5-2011

Bilingualism and math cognition

Michelle M. Guillaume
University of Nevada, Las Vegas

BILINGUALISM AND MATH COGNITION

by

Michelle Melissa Guillaume

Bachelor of Science
University of New Orleans
2002

Master of Science
University of Nevada, Las Vegas
2005

Master of Arts
University of Nevada, Las Vegas
2008

A dissertation proposal in partial fulfillment
of the requirements for the

Doctor of Philosophy in Psychology
Department of Psychology
College of Liberal Arts

Graduate College
University of Nevada, Las Vegas
May 2011

THE GRADUATE COLLEGE

We recommend the dissertation prepared under our supervision by

Michelle M. Guillaume

entitled

Bilingualism and Math Cognition

be accepted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Psychology

Mark H. Ashcraft, Committee Chair

David E. Copeland, Committee Member

Colleen M. Parks, Committee Member

CarolAnne M. Kardash, Graduate Faculty Representative

Ronald Smith, Ph. D., Vice President for Research and Graduate Studies
and Dean of the Graduate College

May 2011

ABSTRACT

Bilingualism and Math Cognition

by

Michelle Melissa Guillaume

Dr. Mark H. Ashcraft, Examination Committee Chair
Professor of Psychology
University of Nevada, Las Vegas

Within cognitive psychology, the fields of bilingualism and math cognition have been investigated relatively separately from one another. Although there has been a substantial amount of research conducted in both areas, few studies have examined mathematical processes as they relate to bilinguals. A couple of the traditional effects found in the math cognition literature, the problem size and associative confusion effects, have been studied with bilinguals; however, bilingual categorization was not carefully controlled for in those studies. There have also been mathematical models applied to bilingual samples; one such model is the encoding-complex model, which has been extended to Chinese-English bilinguals. The current study aims to examine math cognition effects after careful control of bilingual categorization has been taken. Tasks will include number naming, simple arithmetic production, and simple arithmetic verification utilizing associative confusion problems with Spanish-English and English-Spanish bilinguals. The study will also examine whether or not the encoding-complex model can be successfully extended from Chinese-English bilinguals to Spanish-English and English-Spanish bilinguals.

ACKNOWLEDGEMENTS

I would like to thank my committee members: Dr. David Copeland, Dr. Colleen Parks, and Dr. CarolAnne Kardash. Your enthusiasm and support for my project helped to keep my motivation high, and it was a pleasure to work with all of you. I want to also say thank you to my math cognition lab partners: Jeremy Krause, Rob Durette, Alex Moore, and Nathan Rudig. Even though there were several ups and downs (and mood swings) throughout the years that we have worked together, I felt that I could always turn to any one of you for patience, help, and encouragement to keep on pressing through. You guys will always have a special place in my heart.

I want to take the opportunity to say a special thanks to my committee chair and advisor over the last six years, Dr. Mark Ashcraft. From the beginning and through til the end, you have helped me grow and learn. I always felt complete support from you; even during the times when my spirits and motivation were low, you were always there to say what needed to be said to get me back on track. Your understanding and wisdom helped me to achieve what I set out to achieve, and I cannot imagine having accomplished this without you.

Finally, a heartfelt thank you goes to Norval and Linda Jessogne. There have been some very difficult times for me along this journey, and I have always felt unconditional love and emotional support from you both. Thousands of conversations have taken place over these years, and I have taken away something valuable from each and every one. Saying thank you a thousand times would not be enough. I love you both with all of my heart, and I would like to dedicate this accomplishment to you.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

In its infancy, the bulk of language research in cognitive psychology was done with monolingual participants, individuals who only read and spoke in one language. As the research area grew, cognitive psychologists became interested in researching language with respect to bilinguals, individuals who read and spoke in two languages. Several questions regarding the representation of each language surfaced. Was each language stored in its own area of the brain, or did both languages share one lexicon? If there were separate stores, how did the individual translate from one language to another? What did the connections look like between languages? These questions and more fueled the fire for bilingual research to begin taking place. Throughout the language research with bilinguals, several models, which will be discussed in the body of this paper, have been created with respect to how two languages function together in the brain. Bilingual language research initially only focused on experiments dealing with verbal materials and imagery; however, some bilingual researchers began to use ideas from the math cognition literature within cognitive psychology to help answer some of the lingering questions regarding bilingual language processing.

It wasn't until the 1970's that cognitive researchers became interested in the field of math; however, after a few breakthrough studies (i.e. Ashcraft & Battaglia, 1978; Groen & Parkman, 1972), math cognition became a growing area of research. For the next 30 years, researchers studying math cognition looked at the different processes involved in addition, subtraction, multiplication, and division. Several common effects were found across studies and will be discussed in a later section. Overall, with the exception of a few scattered studies in the 1960s and 1970s (i.e. Kolers, 1968; Marsh & Maki, 1976), math cognition and bilingual language research remained relatively separate in the field of cognitive psychology. Not until the 1980s, when the work of Chomsky (1986) suggested a connection between linguistic and numerical knowledge, did bilingual research utilizing mathematics begin to flourish, and that is where the topic of this dissertation experiment was focused.

Before discussing this dissertation experiment and its results, a detailed review of the literature will be conducted. The first section will discuss some necessary terminology and methodological considerations for using bilingual participants. The second section will then cover cognitive research that has been conducted using bilingual participants, including a discussion of the different models that have been proposed for bilingual memory representation. In the third and fourth sections, mathematical cognition research, specifically, those areas pertaining to mental representation and mathematical performance, will be discussed. The fifth section will look at the research that has been conducted to date using traditional mathematical tasks to further the study of bilingual memory representation and mathematical processes. Finally, the experiment for this dissertation project will be explained, results will be given, and a detailed discussion will follow

CHAPTER 2

LITERATURE REVIEW

Methodological Considerations Used in Bilingual Studies

Individuals who have acquired two languages, bilinguals, provide very useful information for cognitive psychologists. Using bilinguals as participants, researchers are trying to hypothesize and make predictions concerning how each language is represented in the brain, whether the two languages share all or some of the same space, how the languages are connected to each other, and if there is a difference, for example, between bilingual processing of verbal and numerical information. While trying to investigate one or more of the above ideas regarding bilingual language effects, it is important to first become familiar with some of the terminologies that are commonly used as well as some important methodological considerations to keep in mind when working with bilingual participants.

Language Terms

When studying bilinguals and mathematics, participants are usually referenced by the two languages that they speak. For example, English-Spanish bilinguals are different from Spanish-English bilinguals. The first language listed, L1, refers to the preferred language, P, of doing mental calculation, and the second language listed, L2, refers to the nonpreferred, NP, language for doing mental calculation. Therefore, in the example above, English-Spanish bilinguals prefer to do mental calculation in English and Spanish-English bilinguals prefer to do mental calculation in Spanish. It is important to note that the meaning of preferred language can change from study to study, and the reader should keep this in mind before interpreting results from bilingual studies. For example, with reference to initial studies of bilinguals using verbal tasks (e.g. picture naming and lexical decision), the preferred language, P or L1, referred to the language that the participants were most comfortable using on a daily basis, probably the language they spoke at home. However, when bilingual participants became of interest to cognitive psychologists studying math cognition, the term 'preferred language' started to refer to the language that the participants preferred to do mental calculations in, not necessarily the language spoken at home. Therefore, it will be important to distinguish preferred language among the different literature reviewed in this paper, depending on whether the study was done using verbal or mathematical stimuli.

Bilingual Categorization

There are different categorizations of bilinguals based on their knowledge and experience with L2. Balanced bilinguals are generally considered to be fluent in L2, possibly having been taught both languages at the same time during childhood, although in some cases, bilinguals may be considered balanced if their experience with L2 spans a very long time period, and if they have been in working and/or living situations where they have predominantly used L2. Unbalanced bilinguals vary in their level of experience with L2. They may have acquired L2 after adolescence or in school, and their years of experience using L2 tend to be fewer. For experimental purposes, it is important to carefully distinguish between balanced and unbalanced bilinguals by collecting several types of information from the participants. Degree of bilingualism has been measured using several self-report inventories; for example, a 9 point self-report scale dating back to Macnamara (1967) has been used, with 1 and 9 indicating use of only one language and 5 indicating equal use of the two languages. Other self-report measures that have been used to assess language balance collect some basic information regarding the ages at which the participant was exposed to both languages and the percentage of time that they speak the dominant cultural language. Participants are then asked whether their ability to speak the first language is better than or equal to their ability to speak the second language.

The literature has not come to a consensus on exactly which measure to use for determining degree of bilingualism; however, Altarriba and Basnight-Brown (2007) examined several bilingual studies conducted between 1989 and 2001. Noting the variability among the studies with regard to the variable of degree of bilingualism, the authors suggested that to achieve a better assessment, an experimenter should collect biographical data, language history data, and language proficiency data. These data would include information such as age of acquisition of both L1 and L2, where and when each language has been used most, proficiency measures in reading, writing, speaking, and listening, and possibly an online assessment of proficiency, such as picture-naming or a translation task (Altarriba & Basnight-Brown, 2007). However, it is important to note that the authors of that article specifically looked at studies using semantic-priming tasks and translation tasks, which contained all verbal materials. No bilingual studies that used numerical stimuli or mathematical stimuli were considered.

Although Altarriba and Basnight-Brown (2007) have valuable points and advice regarding the assessment of bilinguals, the advice given required several different types of measures to collect language background information. To administer several different types of measures may require participants to come in for more than one session: one session to collect language information for categorization and another session to collect data for the research question at hand. A second group of researchers, Marian, Blumenfeld, and Kaushanskaya (2007), developed the Language Experience and Proficiency Questionnaire (LEAP-Q), which has come to be commonly used for the assessment of bilingual categorization. In their 2007 article, the LEAP-Q was tested for internal validity and criterion-based validity. Results of two studies found the LEAP-Q to be an effective tool for assessing bilingual language status. The questionnaire only takes 15 minutes to complete for two languages and adds five additional minutes for each additional language being assessed. Three main variables of interest on the LEAP-Q are language competence, language acquisition, and prior and current language exposure. Overall, the LEAP-Q seems to be the best measure to use for establishing bilingual categorization.

Other Considerations

There are also a few other considerations that need to be taken into account before beginning a bilingual study. One consideration is that of word length; if the length of words being used in one language is significantly different than the word length in the second language, the data collected may not be as reliable or straightforward to analyze participants' response times, which are likely to differ if the words being used as stimuli are of differing lengths. Also, as noted by Altarriba and Basnight-Brown (2007), it is a possibility that word length may not only affect reaction times, but also word processing, recognition, and pronunciation. It is therefore important to try to control for word length when designing a bilingual study, and, in the case that languages do differ greatly in word length, it is important that it be noted in the interpretation of the study. Of the studies dealing with bilinguals and mathematics (Duyck & Brysbaert, 2002; Duyck & Brysbaert, 2004; Frenck-Mestre & Vaid, 1993; Marsh & Maki, 1976; Spelke & Tsivkin, 2001; Vaid & Frenck-Mestre, 1991), none mentioned whether number word length was controlled for between L1 and L2. This is especially of concern for a couple of reasons. Firstly, the results of three studies conducted by Ellis (1992) found that number word-lengths had an effect on participants' mental calculation and counting. Secondly, with the exception of Vaid & Frenck-Mestre (1991), which analyzed percent recall, reaction time was the main dependent variable being considered in all of the above studies. It is also important to note here that, along with error rates, reaction time is the main dependent variable considered in the bulk of the math cognition research. A summary of the results of Ellis (1992) are given below.

Specifically, in experiment 1, Welsh and English children were given different types of arithmetic problems to calculate mentally; four types of problems included simple multiplication (e.g. 5 x 3), simple multiple-figure addition (a three digit number plus a two digit number e.g. 305 + 42), complex multiple-figure addition (a three digit number plus a two digit number involving the carrying operation e.g. 134 + 88) , and multiple figure addition (adding nine single digits e.g. 5 + 3 + 7 + 4 + 9 +8 + 6 + 5 + 3). Although significant latency differences were not found between languages for problem types 1 and 2, results showed that the English children solved problems significantly faster than the Welsh children for problem types 3 and 4. Problems that required more steps and storage were both prone to longer latencies and more errors for the Welsh children. To demonstrate that the results obtained were due to number word length in each language and not due to problem difficulty, the author conducted experiment 2 using adults from the community. It was found that number naming (reading number words on cards) and counting out loud from 1 to 100 were significantly slower when performed in Welsh, where the number words were longer, than when performed in English.

Two more considerations when testing bilingual participants are the use of cognates as part of the stimuli and cultural factors. Cognates are words that are similar in spelling, pronunciation, and meaning across languages. For example, in both French and English the number word for six is spelled the same way, which could facilitate reaction times in both languages; it was for this reason that the number word for six was excluded from display in Frenck-Mestre & Vaid (1993). Finally, depending on the languages being tested, there are cultural factors to be considered, especially when using mathematical and number word stimuli. For example, studies examining Asian and North American cultures have found significant differences with regard to mathematical abilities. Although not a bilingual study, Campbell and Xue (2001) found simple arithmetic performance differences between Chinese and non-Asian North Americans, with the Chinese greatly outperforming non-Asian North Americans. The authors could not find anything in the data to attribute to the more efficient retrieval skills and less frequent usage of procedural strategies of the Chinese and therefore, attributed the superior performance to culture-specific factors. One could speculate that it is possible that different cultural groups might differ on their emphasis of math, their way of teaching math, and/or when they start teaching math. Therefore, it would be important to consider such cultural differences in performance when conducting bilingual research.

Cognitive Research with Bilinguals

Most of the cognitive research concerning bilinguals has been focused on developing models and theories around how each of their respective languages are represented and organized in memory. The models focused on a variety of concepts including word mapping, visual word recognition, auditory word recognition, and lexical and semantic storage and organization. In the beginning, bilingual memory models were based solely on experiments examining verbal information. Since the 1980s, several models have been proposed. All of the original models were based on experiments using verbal tasks such as lexical decision, semantic priming, and translation tasks.

Potter, So, von Eckardt, and Feldman (1984), proposed two models to describe how bilinguals map words to concepts in their first and second languages; both models posited separate lexical stores for each language, but only one conceptual store. The difference between the models was whether only one or both languages had access to the conceptual store. One model was a word association model (Appendix 2, Figure 1), where only L1 had access to concepts, and words given in L2 would have to be first translated to L1 before access to the concepts would be given. The second model was a concept mediation model (Appendix 2, Figure 2) in which both L1 and L2 lexical stores had access to one common conceptual store through separate links. It was subsequently found, using both translation and picture naming tasks with bilinguals, that the performance of less fluent bilinguals fit better with the word association model, and the performance of more fluent bilinguals fit better with the concept mediation model.

There have also been attempts at combination models positing one semantic store for both languages but separate lexical stores for each language. The predominant model was the revised hierarchical model of bilingual memory representation (RHM), (Appendix 2, Figure 3), (Kroll & de Groot, 1997). The authors built on evidence from Potter et al (1984), which indicated a switch from only lexical to conceptual access as the bilingual individual became more fluent in the second language. Still a concept mediation model, the RHM combined Potter's word association and concept mediation models; connections were made between L1 and L2 as well as between L1, L2, and concepts. Because L2 was learned later, the lexical connections from L1 to L2 were weak, but the lexical connections from L2 to L1 were strong; initially, unbalanced bilinguals need to translate everything from L2 to L1 to access the conceptual store. The connection between L1 and the conceptual store was also stronger than the connection between L2 and the conceptual store; however, as the bilingual would become more fluent, there was room in the model for the connections to strengthen. With that in mind, it made sense to frame the model with varying strengths of lexical and conceptual connections between both L1 and L2 (Kroll & de Groot, 1997). Support for the RHM was found in Kroll and Stewart (1994) in which fluent Dutch-English bilinguals translated words in either semantically categorized or random lists. Results indicated faster translation from L2 to L1 than from L1 to L2. It was also found that semantic context only affected translation from L1 to L2 (Kroll & Stewart, 1994; see also Sholl et. al ,1995).

Other models include the Bilingual Interactive Activation (BIA) model (Appendix 2, Figure 4), (Dijkstra & Van Heuven, 1998) and the BIA+ model (Appendix 2, Figure 5), a revised version of the BIA model (Dijkstra & Van Heuven, 2002). Both models were inspired by the Inhibitory Control (IC) model specified by Green (1998) as well as the Interactive Activation (IA) model developed in McClelland and Rumelhart (1981; Rumelhart & McClelland, 1982). The models assume a non-selective language framework suggesting parallel processing. Also, both models were word recognition models in which the primary goal was to recognize a word as belonging to one language or the other. Unlike the previous bilingual memory models, an advantage of both the BIA and BIA+ models was that they were designed and implemented on the computer, allowing simulation of several experimental studies. The original BIA model proposed a single, non-selective, lexical store. Non-selective indicated that as a word string was coming into the system, both languages were activated at the same time, as opposed to only one language being activated by the word string. The BIA+ model incorporated the BIA model with several adaptations. One adaptation was the addition of a task/decision system to the word identification system; the task/decision system would be set up possibly during practice and would be specific to the activity at hand. Another adaptation from the BIA model was to the language nodes; instead of words being identified on the sole basis of orthographic representations (assumed by the original BIA model), the BIA+ model posited that bilinguals also recognized words as either L1 or L2 words based on phonemic and semantic representations (Dijkstra & Van Heuven, 2002). The BIA+ model theorized that those representations would be activated faster for L1 than for L2, but that the time between activations would not be very quick (assuming fluency in both languages). For details, see Dijkstra & Van Heuven (2002), French & Jacquet (2004), Thomas & Van Heuven (2005).

Finally, using computer modeling, connectionist models that incorporate learning mechanisms have been proposed and implemented to try to examine exactly how the two languages become organized in memory. One of these computer models is the BSRN (bilingual simple recurrent network; French, 1998). The BSRN is a computer model, which learns two artificial languages with different vocabularies. The network learns the structure for each language and then is given new sentences and tries to predict the last word in the sentence. According to simulations, as long as the language switches remain infrequent, the model could develop separate representations for each language, although there would be a lot of overlap for representations (Thomas & Van Heuven, 2005). Another connectionist model is the SOMBIP (Self organization of bilingual memory; Li & Farkas, 2002). This model is designed to capture both bilingual language production and comprehension. The SOMBIP has advantages in that it can account for priming and interference effects and can also examine lexical representation while taking into account differing levels of bilingual proficiency and working memory capacity. Chinese and English are the current languages for which this model is currently active. (For a review of connectionist models of bilingual language organization, see French & Jacquet (2004) and Thomas & Van Heuven (2005)).

To summarize, there have been a number of bilingual memory models proposed; however, the designs have been based solely on verbal representations and language learning, and none of the models have taken mathematical processing specifically into consideration. However, there have been several studies outside of the modeling works that have examined bilingual individuals performing math related tasks. Before those studies are mentioned, it is important to discuss some of the main ideas and findings from original math cognition studies.

Mathematical Cognition

Common Effects Found in the Literature

Since the 1970s, the math cognition field has grown, and the research areas within the field have expanded from experiments using simple arithmetic stimuli to investigate differences among mathematical processes to experiments aimed at developing theories about just how numbers are represented in the brain. Four main effects that have been found in monolinguals throughout the mathematical cognition literature include the problem size, distance, split, and confusion effects. Each effect will be described below, along with some relevant literature in which that effect was demonstrated.

The problem size effect refers to longer reaction times being observed for arithmetic problems as a function of increasing operands and increasing sums. This effect has been robust and found throughout the math cognition literature for both production and verification tasks as well as for addition, subtraction, multiplication, and division (for review, see Zbrodoff & Logan, 2005). Distance effects have been observed in number comparison tasks throughout the math cognition literature. When comparing two digits, judgments are made faster when the distance between the two digits is greater. Numbers that are closer in magnitude, such as 3 and 4, are harder to discriminate than numbers that are further apart, such as 3 and 9. Original evidence of the numerical distance effect was found in Moyer and Landauer (1967); however, many studies since then have been able to demonstrate this effect for pairs of both Arabic digits and number words (e.g. Dehaene & Akhavein, 1995).

In math cognition, for false problems, the term *split* refers to the numerical distance between the true answer to the problem and the false answer presented to participants. The split effect occurs when reaction time latencies increase as the split gets closer to the true answer. Therefore, small splits (e.g., ±1 or 2), will have longer reaction times than large splits (e.g. ±8 or 9). This suggests that large splits are easier to dismiss as implausible. Pioneering work demonstrating the split effect was shown in Ashcraft and Battaglia (1978). In that study, the split effect was manipulated in the answers of the false stimuli presented such that some of the false answers were different from the true answer by ±1 or 2 (termed reasonable false) and other false answers were different from the true answer by ±5 or 6 (termed unreasonable false) (see also Ashcraft & Fierman, 1982; Ashcraft & Stazyk, 1981).

Associative confusion effects have also been demonstrated in math cognition. Confusion problems are false problems (i.e., the answer provided with the problem is technically false); however, the answer would be considered true under a different operation. An example of this would be the false addition problem of "4 + 3 = 12". This is a confusion problem because while that answer is false, if 4 and 3 were multiplied instead of added the correct answer would be 12. Other types of confusion problems have been used for multiplication problems in which the answer provided was a multiple of one of the two problems digits such as the problem "6 x 5 = 25" (Stazyk, Ashcraft, & Hamann, 1982). Confusion problems have resulted in longer reaction times and higher error rates compared to non-confusion problems (Hamann & Ashcraft, 1985; Winkelman & Schmidt, 1974).

Cognitive Number Processing

So far, this section has focused on the prominent effects found throughout the math cognition literature. It will also be important to examine how math cognition researchers have theorized exactly what is going on when individuals are performing arithmetic operations as well as the theories describing exactly how numbers are represented and organized in the brain. Many ideas have been presented regarding the mental representation of numbers and arithmetic processes.

Initial work focused on developing models for how mental arithmetic was performed. Some of the first studies were conducted by Parkman and Groen (1971) and Groen and Parkman (1972). Results from those studies led the authors to develop a "count by min" model for children performing simple addition. According to the "count by min" model, a first grader would solve the problem X + Y = ? in the following manner: first, a mental counter would be set to the larger of the two addends (max(X,Y)). The child would then count up by the minimum addend (min(X,Y)) one step at a time to achieve the answer. For example, given a problem such as 5 + 2, the child would hold the larger addend, 5, in memory, and then increment by 1s until the number of increments equaled the minimum addend, 2. Reaction times for adults were significantly faster than for children, and adult data also showed evidence of a problem size effect. Results lead the authors to theorize that, for adults, simple addition was mostly an automatic process; however, for an unknown proportion of simple addition problems, adults would revert back to the counting model used by children, as illustrated by slower reaction times for addition problems with larger addends (Groen & Parkman, 1972).

A strictly counting model for adults was not accepted by some researchers at that time. Ashcraft & Battaglia (1978) conducted two experiments that provided evidence contradictory to a strictly counting model for adults. Instead, the authors posited a network retrieval model for simple addition. The network representation for addition was a square with the digits 0-9 on two adjacent sides and the sums located at the intersection point of any two numbers. Incorporating the exponential problem size effect, modifications to the square were presented that included stretching out the distance between larger sums or making the distance between entry sums larger as the addends became larger (Ashcraft & Battaglia, 1978). A similar structure for a network retrieval model of multiplication was also proposed after evidence of the problem size effect was found (Stazyk, Ashcraft, & Hamann, 1982).

By the late 1980s and early 1990s, some of the focus in math cognition switched from models of arithmetic to the exact mental representation of number. Firstly, it is important to point out that individuals can understand numbers in at least two forms: Arabic digits (4, 5, 6) and number words (four, five, six) (Dahaene, 1992), and this could be doubled in the case of bilinguals whose second language may also have different characters to represent numbers. This is important because results can differ depending on the type of numerical stimuli used. Three main models of cognitive number processing have been proposed: The abstract-code model (McCloskey, Caramazza, 1985; McCloskey, 1992; McCloskey & Macaruso, 1994, 1995), the encoding-complex model (Campbell & Clark, 1988; Campbell & Epp, 2004), and the triple-code model (Dehaene, 1992; Dehaene & Akhavein, 1995).

Firstly, a model for cognitive number processing and calculation was introduced by McCloskey & Caramazza (1985). According to the model, there were two independent systems working to process numbers: a calculation system and a number processing system, which included components for number-comprehension and number-production. The calculation system was specified to contain three components: processing of operational symbols, retrieval of arithmetic facts, and execution of calculation procedures. The number processing system was also further broken down so that both the production and comprehension subsystems of the number processing system had components for distinguishing between Arabic digits and verbal number words; furthermore, each of those components contained mechanisms for lexical and syntactic processing (McCloskey & Caramazza, 1985). According to the model, in order for numerical comprehension, production, and calculation to take place, numbers needed to first be translated into an abstract semantic representation. The model has therefore been referred to as the abstract-code model (Campbell & Epp, 2004). A schematic of the model can be found in Figure 6 of Appendix 3. Although the authors used only data collected from several brain damaged patients to provide evidence for the model, they believed it to generalize to normal populations as well. In sum, the model would take in numerical information and translate that information into an abstract code that the calculation system would then work with. As soon as calculations were complete, the information would then be translated from the abstract code back into a numerical representation that could then be produced by the number production system.

After describing the initial model in detail, an additional model, based on data from two brain damaged patients, was proposed to specifically examine spoken verbal-number production (McCloskey, Sokol, & Goodman, 1986). According to this model, on the basis of semantic input, the model would produce a syntactic frame specifying the number to be produced in terms of number-lexical class and position within class (e.g., TENS: {3}).

Secondly, in response to the model proposed in McCloskey, Sokol, and Goodman (1986), Campbell and Clark (1988) published their own encoding-complex model of cognitive number processing (for a bilingual version, refer to Appendix 3, Figure 7). According to this view, numbers, whether presented in Arabic or number word format, were not being translated into an abstract number code for further calculation or processing. The encoding-complex view posited a more integrated network that would mediate number production, comprehension, and calculation in a more parallel fashion, without any transfer to an abstract representation before doing so. The model assumes specific representations for number processing, which are associatively connected to create the possibility that an activation of one representation could, at the same time, activate other representations within the network. Also, each representation could be involved in production, comprehension, and calculation functions (Campbell & Clark, 1988). So, according to this view, an Arabic digit may activate several codes simultaneously within the network i.e. both the numeric digit and the number word representations. Activation of any specific representation would depend on incoming information from visual, semantic, associative, and procedural connections to help choose from the multiple active representations (Campbell & Clark, 1988). The encoding-complex model has been supported by several research studies that have evidence for multiple numerical procedures being activated in parallel using both production tasks (Campbell & Graham, 1985) and verification tasks (Stazyk et al., 1982; Zbrodoff & Logan, 1986).

Thirdly, the triple-code model of numerical number processing (Appendix 3, Figure 8) was proposed in Dehaene (1992). According to Dehaene (1992), in addition to mental calculation, individuals also are able to quantify and approximate, separate from symbolic manipulation. With respect to quantification and approximation, it has been posited that children and adults convert numerals into an internal magnitude code or number line (Dehaene et.al, 1990). This facet of the triple-code model is similar although more specific than the general abstract representations proposed in the abstract-code model. Also, this mental number line seemed to extend horizontally from left to right, with smaller numbers on the left side and larger numbers on the right side. Evidence for this was found by Dehaene, Bossini, and Giraux (1993). This study had participants make parity judgments (whether a digit was odd or even); importantly, participants had to press one key with the right hand if the number presented was even or a separate key with the left hand if the number presented was odd. Results indicated faster responses to small numbers with the left hand and faster responses to large numbers with the right hand. This finding of a spatial-numerical association of response codes was termed the SNARC effect (Dehaene et. al, 1993). Although the abstract-code model and the encoding-complex model had been proposed, Dehaene and colleagues found them to be incomplete since neither model contained details of approximation or quantification. These elements were included in the triple-code model posited by Dahaene (1992). According to the triple-code model, numbers could be represented in three different ways: (a) an auditory-verbal code, (b) a visual-Arabic code, or (c) an analogue magnitude code. Numerical procedure could be tied to specific input and output codes. For example, the auditory-verbal code contributed to written and spoken numerical input and output while also providing a representational basis for simple addition and multiplication facts. The visual-Arabic code was responsible for digital input and output as well as multi-digit operations. Finally, the analogue magnitude code supported approximate calculations, estimation, numerical comparisons, and possibly subitizing, the rapid, accurate, and confident judgment of number found for small numbers of items (Dehaene, 1992; Dehaene & Cohen, 1991).

In an effort to gain evidence for the triple-code model, Dehaene & Akhavein (1995) first attempted to answer some questions regarding mathematical performance given Arabic digits versus number words. In experiment 1, participants were given pairs of numbers and asked to respond ‘same’ or ‘different’ to whether the two numbers represented the same numerical magnitude. The number pairs were presented in Arabic form (4 and 4), verbal form (four and four), or mixed form (4 and four or four and 4). Results showed a significant distance effect for difference judgments on all types of stimuli. The closer the two numbers were on the mental number line, the slower the participants were to decide ‘different’. This suggested semantic access was occurring regardless of presentation format (Dehaene & Akhavein, 1995). Both experiments 2 and 3 yielded similar results and support for semantic access.

Upon close examination of the number processing models described above, many similarities could be drawn among those presented and the models of bilingual memory described in the section covering cognitive research with bilinguals. Before looking at those similarities, it is important to first understand if any of the number processing models has anything to say with regard to bilinguals and their numerical representation systems. The next section will look at each of the number processing models described above, and what, if anything, has been said with respect to each model applying to bilingualism.

Cognitive Number Processing Models and Bilingualism

With regard to the McCloskey and Caramazza (1985) and McCloskey, Sokol, and Goodman (1986) abstract code and verbal number processing models, the authors addressed number processing in other languages, and mentioned that the model proposed for verbal-number production was designed specifically with the English verbal-number system in mind. Although the article could not speak to whether the model specifics could be applied to those persons who spoke languages in addition to English, it was mentioned that the basic principles of the model could possibly generalize to other languages; however, no articles pertaining to research applying those models to other languages or bilinguals could be found. Also, although Dehaene and colleagues have done work examining bilinguals and mathematical processing using such technology as fMRI to examine the difference between exact and approximate calculations in bilingual brains (e.g. Dehaene et al., 1999), there has been no work specifically tying the triple-code model to bilingual number processing or bilingual calculation.

On the other hand, the encoding-complex model, proposed in Campbell & Clark (1988) has successfully been extended to Chinese-English bilinguals in Campbell and Epp (2004). Unlike the abstract-code and triple-code models, which assumed processes underlying numerical cognition were not affected by numerical surface forms, the encoding-complex model functioned under the theory that numerals activate a network of associations, including both relevant and irrelevant information (Campbell & Epp, 2004). According to the article, Chinese-English bilinguals performed a number of math related tasks, including number naming, number comparison, and simple multiplication and addition problems. Performance did indicate a more integrated approach for bilinguals' numerical cognition than provided for in the previous abstract-code and triple-code models, which assumed a more additive approach to number processing. In order to extend the model, the encoding-complex model combined the representational assumptions of the triple-code model (Dehaene & Akhavein, 1995) with the assumption that efficiency of processing can vary with format (Campbell & Epp, 2004).

It may also be important to consider possible cultural implications when examining bilinguals and mathematical performance. Campbell and Xue (2001) found significant differences between Asian and non-Asian samples for performance on mathematical tasks. Although no specific cultural factors were mentioned in that article, there have been differences found that should be mentioned with regard to the differences between Asian and non-Asian mathematical competencies. For example, when learning number words in a native language, if we compare Chinese and English, there are no rules to apply to learn the number words for 1-10; they must simply be memorized. From 11-19 however, the languages take different paths to number naming. In Chinese, the words are mapped directly onto the Hindu-Arabic number system, making it easier for a child learning these numbers to create a rule to better remember the names (Miller, Kelly, & Xiaobin, 2005). On the other hand, English names for the numbers 11-19 are much more complicated and do not follow any specified rules that children learning the numbers would be able to apply to more readily remember them. Above 20, English number words become a lot more regular; however, the differences between English and Chinese from 11-19 could be the beginning of developmental performance differences between children. English children may become "stunted" in a way as they try to learn and progress through the number words with no clear rules to follow until twenty. This was demonstrated in a longitudinal study by Miller, Smith, and Zhang (2004) where Chinese and English children were followed monthly and asked to recite the number list. At age 2, there was not a significant difference between the English and Chinese children, with both groups able to count to ten fairly easily; however, by the end of age 3, Chinese children were able to count to 60, whereas English children could just barely get past 20. This result compounded by age 4, where Chinese children could readily count to 100 by the third month, and English children could not get to 40 by the end of their fourth year. Similar difficulties can be found with the verbal words for both ordinal and rational numbers, which have discernible language rules that are easy to follow in Chinese but have to just be memorized in English. So it seems that differences between Chinese and English performance on mathematical tasks possibly begins developmentally when children are first learning number words.

Miller, Kelley, and Xiaobin (2005) also point out some distinct cultural differences between Chinese and English parents with regard to mathematics. According to the literature (Stevenson & Lee, 1998; Kelley, 2002) (a) U.S. parents spent more time preparing their children for school with reading activities than math, compared to Chinese parents who balance the two subjects more equally, (b) U.S. parents are more likely to attribute their children's successes and failures to innate factors rather than effort, and (c) There is a general lack of communication between home and school in the U.S. (Miller, Kelley, & Xiaobin, 2005). Together with the language factors mentioned above, there are some very important cultural differences that could be exerting great influence over the differences in mathematics performance between the Chinese and U.S. populations beginning when children first begin to learn numbers prior to formal schooling and continuing into and throughout formal schooling.

Summary

After reviewing the math cognition literature and examining some of the pertinent numerical cognition models, it can be seen that some of the same questions asked in the bilingual literature regarding L1 and L2 have been asked regarding Arabic digit versus number word processing in the mathematical cognition literature (Dehaene & Akhavein, 1995). For example, in the previous section on cognitive research with bilinguals, it was shown that similar arguments have been made about whether a bilingual has both L1 and L2 stored in one common lexicon or two separate lexicons with a common connection to a conceptual store, or whether L1 and L2 have direct communication. Arguments have also been made within the math cognition literature saying that number words and Arabic digits are processed along two different pathways, and then converge into one input system. Other issues in the mathematical cognition literature that have paralleled the bilingual literature concern whether there is any direct access between the verbal and Arabic digit lexicons that bypasses any semantic access as well as if there is a preferred notation or "language" for numbers. If there is a preferred "language" for numbers and stimuli are presented in the nonpreferred "language," is there a translation process that needs to take place before any verifications, choices, or calculations can be made for a particular task? Some direct comparisons can be made between the bilingual memory models and the cognitive number processing models.

The bilingual memory models have looked at both comprehension and production as separate processes within the bilingual lexicon, and the numerical cognition models have looked at the same idea with regard to numerical production and numerical comprehension. The Revised Hierarchical Model of bilingual memory (RHM) proposed connections of varying strengths between L1 and L2 and a conceptual store; it is possible that there are also connections of varying strength within the bilingual lexicon between L1 and L1 number words, Arabic digits, and arithmetic fact retrieval. The Bilingual Interactive Activation Model (BIA) and its latter version, the BIA+ model, proposed parallel processing for both languages in an identification system, which, for the BIA+ model, fed into a task schema containing decision criteria for choosing which language was to be used. That idea is not unlike the integrated processing proposed in the encoding-complex model of cognitive number processing.

In the next section covering mathematical cognition research with bilinguals, several studies will be discussed, some of which show evidence of a preferred language for bilingual individuals performing mental calculations similar to the idea of a preferred notation for number in the math cognition literature. Other studies will also be discussed that tried to examine some of the major findings and effects from the math cognition literature using bilingual samples.

Mathematical Cognition Research with Bilinguals

Math cognition research has taken two directions in reference to bilinguals and math. One direction has been more theoretical in nature. Some studies in this direction have used math tasks to try to investigate numerical representation in both languages. Other studies have tried to either lend evidence to or disconfirm some of the models of bilingual memory mentioned in the previous sections.

Spelke and Tsivkin (2001) conducted three experiments to investigate the idea of numerical representation using exact and approximate number calculation for bilingual Russian-English participants. In Experiment 1, participants were trained on exact large number addition, exact addition in different bases, and approximations of cube roots and logarithms. Training and testing took place in both languages. During testing, problems were presented in number word format on a computer screen with two possible answers displayed in number word format below the problem just right and left of center. Results indicated that approximations, at least of cube root and logarithm problems, seem to draw on representations independent of language as indicated by reaction time results i.e. approximations were solved with equal speed on both trained and untrained problems. However, for the exact addition both of large numbers and addition in different bases, participants responded faster in the language of training, either English or Russian, possibly pointing to exact number facts being represented in a more language-specific form.

Experiment 2 was conducted to further investigate the results of experiment 1 and examine whether exact, but not approximate, number representations were independent using only exact and approximate addition and multiplication (Spelke & Tsivkin, 2001).

No addition in different bases, logarithms, or cube roots were used. For exact addition, participants performed faster in the trained, than in the untrained language, and for approximate addition, they performed with equal speed. Due to high error rates and greater variability in the results, multiplication data were harder to analyze and draw conclusions from. Although inconclusive, the multiplication data showed similar reaction times for multiplication problems on both exact and approximate problems as well as faster reaction times in the trained compared to the untrained language. Since there was no evidence of language-dependent representation for multiplication, the authors used both the addition and multiplication data to support Dehaene's (1997) theory that non-verbal number representations were accessible to addition, but not to multiplication (Spelke & Tsivkin, 2001). Therefore, using only the exact and approximate addition data, the implications were once again that exact number representations were language-dependent and approximate number representations were language-independent. On a number line, exact number representations would appear clear and distinct, but approximate number representations would appear as blurs.

In experiment 3, participants were given history and geography lessons containing numerical and non-numerical information. Participants were required to read the lessons aloud, in silence, and listen to the experimenter read them, two readings each; numbers were presented in the lessons as number words, in the appropriate language. Three training sessions were given in either Russian or English and one testing session was given in both languages. Once again, participants responded faster in the trained language to exact number questions (i.e. age of a character), indicating a language-specific component for exact number quantities. All three experiments provide evidence that small, exact numbers and large, approximate numbers can be represented independently of language, and that representations of only exact large numbers depend on a specific language with a counting system.

In addition to evidence for language dependent exact and approximate mental representations of numbers (Spelke & Tsivkin, 2001), one study produced some results for which the Revised Hierarchical Model of Bilingual Memory (RHM), refer to figure 3, could not account (Duyck & Brysbaert, 2004). Four experiments investigated assumptions of the previously hypothesized RHM. Dutch-French bilinguals were asked to name Arabic digits as well as Dutch and French number words. Translation was separated into forward and backward translation; in forward translation, the naming language was French (i.e. Dutch number word given, name it in French), and in backward translation, the naming language was Dutch (i.e. French number word given, name it in Dutch). A number magnitude effect (e.g. a smaller number like 2 was easier to translate than a larger number like 8) was found in both forward and backward translation. This effect was not consistent with the current Revised Hierarchical Model (RHM), which predicted a number magnitude effect only in the forward translation condition, from Dutch to French; according to the RHM, the semantic connections between the number words and the corresponding Arabic digits would be strong from L1 to L2, but weak from L2 to L1; however, Duyck and Brysbaert (2004) found no difference between the semantic connections as seen in the number magnitude effect being found from both L1 to L2 and from L2 to L1.

In an effort to see whether results were due to unbalanced bilinguals already being very proficient in the second language, participants were told that they were learning the

Estonian number words for 1-15; however, the actual words that the participants were learning were fabricated by the experimenters. During the experimental phase, which was identical to the experiments above, participants were asked to say the name of the number presented (Arabic digit or number word) in either Dutch or Estonian. Even for just-learned number words, a number magnitude effect was found for both forward and backward translation, suggesting that learned (number) word forms are mapped onto existing abstract (magnitude-related) semantic information very early in the L2 acquisition process (i.e. a mental number line).

A second direction of the math cognition research has put the models of bilingual memory aside, and has tried to examine bilinguals on a number of math-related tasks to try to examine several other effects. These include the word length effect, the preferred language effect, and the compatibility effect. The word length effect occurs when the number words of one language are longer than in the other, resulting in slower reaction times even when just naming numbers. The preferred language effect refers to bilinguals performing faster in their preferred language (i.e. the language they prefer to do mental calculation) than in their second language. Finally, the compatibility effect refers to reaction time facilitation when the presentation format of the numbers is the same as the format of the response (e.g. the participant is presented with a Spanish number word and asked to respond in Spanish). Other questions revolve around exactly what differences occur, if any, when bilinguals are forced to do mathematics in both of their acquired languages.

The word length effect has been well documented for bilinguals with regard to performance on verbal tasks; however, a natural question was whether this word length effect would carry over into numbers and mathematics because the mental representation of numbers and arithmetic processes is so heavily debated. Ellis (1992) conducted three experiments to investigate the word length effect on number words in Welsh-English bilinguals. Both children (between the ages of 9 and 12) and adults were tested. Children were asked to compute the answers to four different types of problems: a simple multiplication, an addition of a three digit number and a two digit number, an addition of a three digit number and a two digit number with carrying, and addition of 9 single digits. Results indicated that English children solved sums faster than Welsh children for types 3 and 4 sums, which involved carrying.

Adults were asked to read numbers on cards as well as to count from 1 to 100 as quickly as possible. The numbers on the cards were determined by giving an adult participant a set of problems, and having him calculate them out loud. The numbers generated at all stages of the calculation were transcribed onto the cards. For example, a type 1 sum such as 7 x 3, was transcribed onto the card as '7 3 21,' corresponding to 7 times 3 equals 21 (Ellis, 1992). Adult participants were shown the transcripted cards and asked to read the numbers out loud; half of the adults were asked to read in Welsh, and half in English. Results showed that bilingual adults were significantly slower to read the numbers on the card and count to 100 in Welsh than in English, indicating a word length effect.

Finally, a field study was conducted in which children were given a sheet of problems consisting of addition, subtraction, multiplication, and division and asked to solve the problems in the language in which they preferred to do mental calculation. Children were then assigned to the Welsh or English group accordingly. The average number of errors was significantly higher for Welsh children than for English children.

To summarize, a word length effect was observed with significantly larger reaction times (RTs) for Welsh number words even when the participants' preferred language was Welsh. An interesting anecdotal finding from this study was that one teacher included a note that indicated that even though the students spoke and were taught in Welsh, by far they preferred to calculate in English, possibly indicating the ease of calculating in a language with less complicated number words (Ellis, 1992).

It has been posited that individuals prefer to do mathematics in the language that they learned to do mathematics, the preferred language effect, and a couple of experiments were conducted in the bilingual math cognition research to investigate that as well. An early experiment was conducted by Marsh and Maki (1976). This was one of the first studies examining bilinguals and arithmetic operations to try to see if there would be a reaction time difference between performing calculations in both the preferred, P, and nonpreferred, NP, languages. This study differentiated P from NP language in terms of the language in which the individual learned to do mathematics, P, and the other language, NP. Participants were given one, two, or three step addition problems consisting of two, three, or four single digits. Preceding each block, participants were told in which language to give the answer. The experiment used English-Spanish, and Spanish-English bilinguals. Reaction times were longer when participants answered in the NP rather than the P language; however, there were not significant error differences. This study showed that bilinguals could do arithmetic operations faster in the P than in the NP language.

Two similar experiments to Marsh and Maki (1976) were conducted by McClain and Huang (1982) with Chinese-English and English-Chinese bilinguals, and Spanish-English and English-Spanish bilinguals. The same addition problems were used with the exception of auditory as opposed to a visual presentation. Participants were presented one, two, and three step addition problems auditorily and were asked to answer the problem in the language in which the problem was presented, either Chinese or English. The preferred language effect was found, showing a significant decrease in RT for the preferred language; however, during testing, Chinese and English presentation was intermixed, and participants had to adjust their responses accordingly. In experiment 2, the authors used Spanish-English bilinguals and replicated experiment 1; however, a condition was added where some of the bilingual participants were presented with problems in only one language and answered in only one language during one session and came back for a second session where the problems were presented in the other language and participants then answered all problems in that language. Results indicated that the preferred language advantage was eliminated when bilinguals were asked to come in for two sessions, one session in the P and one in the NP language (McClain & Huang, 1982). The authors theorized that the preferred language advantage was due to both languages needing to be active during one session, and that it was the language switch that was responsible for the longer RTs in the NP language.

Some other interesting questions in math cognition include topics such as number format, digits versus words, and number words in L1 compared to L2. Vaid and Frenck-Mestre (1991) used an incidental recall task to investigate whether Spanish-English bilinguals would retain any information about language format of numbers presented.

Bilinguals were presented visually with 20 numbers written in number word form, half presented in Spanish and half presented in English. Participants were told to write all the numbers down in either all Spanish or all English, so half of the words were being copied and half the words were being translated. At the end, the 20 numbers were presented as Arabic digits, and participants had to try to recall whether the number had originally been presented to them in Spanish or in English. Bilingual participants performed above chance on the incidental recall task, indicating at least some memory about the language format of the number. They also displayed a compatibility effect, meaning that if the number was presented to them in the same language that they needed to write down, they were more likely to recall the correct language format during the incidental recall task. The authors argued that even though there has been a preferred language effect found in the literature for other arithmetic tasks, that effect did not seem to apply with regard to incidental recall of language format for number symbols presented (Vaid & Frenck-Mestre, 1991).

So far, the experiments mentioned, while testing bilinguals using numerical stimuli and some mental calculation, have not tested any of the well-known effects found in the math cognition literature. In 1993, Frenck-Mestre & Vaid conducted two experiments with English-French bilinguals and investigated two well-known effects in the math cognition literature, the split effect and confusion effects. Experiment 1 used a true/false verification task for simple addition problems presented as Arabic digits, English number words, or French number words. Small and large splits were also used, ±1 or 2 and ±5 or 6, respectively. Slower RTs were found for problems presented in L2; however, RTs were slower in general for number words compared to Arabic digits, with the mean verification difference between Arabic digits and number words in L1 being greater than the verification difference between L1 and L2. Small splits showed longer RTs for all three conditions (a traditional split effect); however, there was not a significant difference in the RTs to small splits from L1 to L2. Experiment 2 used multiplication and addition stimuli where the false answers were either numerically related (confusion problems) or unrelated. Results indicated associative confusion effects for L1 only, possibly due to weaker associations in the second language. The authors concluded that language played a role in the retrieval of stored arithmetic knowledge, with bilinguals being at a disadvantage in their second language. Retrieval of arithmetic facts and the automatic spreading of activation within the network of numerical facts was found to be at least language-sensitive, if not language dependent (Frenck-Mestre & Vaid, 1993).

Summary

There has been a significant amount of research dedicated to studying bilingualism. Most of the research has lent itself to creating a well-defined model of bilingual memory representation and organization. With that goal, most of the research has been done using verbal tasks such as the semantic priming task and the translation task.

Overall, there seem to be some consistencies in the literature such as the word length, preferred language, and compatibility effects, even when testing using mathematical stimuli (e.g., Ellis, 1992; Marsh & Maki, 1976; Vaid & Frenck-Mestre, 1991). Traditional effects such as the split effect and confusion effects were also found to be demonstrated by bilinguals performing mental calculations. However, it should be considered that some of the studies mentioned are difficult to directly compare due to drastic language and methodological differences among studies. Most of the studies did not categorize the bilingual participants in the same way or according to the same criteria. Also, the languages studied varied drastically from Chinese to French to Spanish.

A study needs to be conducted in which all of the methodological considerations, as established by Altarriba and Basnight-Brown (2007), for categorizing bilingual fluency are taken into consideration. The LEAP-Q is the current measure of bilingual categorization that has taken many, if not all, of the suggestions proposed in Altarriba and Basnight-Brown (2007). After a more precise categorization of the bilingual participants has been made, then it makes sense to try to replicate some of the math cognition effects found with bilinguals. For example, with carefully categorized bilinguals, a study could be done to illustrate the evolution of the confusion effect. Frenck-Mestre and Vaid (1993) found confusion effects in L1 only and explained that effect in terms of weaker associations for L2. This might only be for really unbalanced bilinguals, and it might be the case that as individuals become more fluent in L2, that the confusion effect will be found for L2 as well. After those effects are investigated, then it would be interesting to look for some of the common effects found throughout the mathematical cognition literature, such as the problem size and split effects. Although some of these effects have been shown among studies using bilinguals and mathematical stimuli (e.g., Frenck-Mestre & Vaid, 1993), it is hard to compare them without proper categorization of balanced versus unbalanced bilinguals.

It may also prove useful to try to extend the encoding-complex model as applied to Chinese-English bilinguals (Campbell & Epp, 2004) to other languages, such as Spanish. Will results turn out similarly even though the two languages do not have different representations of number? Results from these studies could also extend beyond psychology, and possibly be helpful within the education system, especially when more and more students in school systems speak more than one language.

Current Experiment

Previous literature examining mathematical cognition in the context of the bilingual lexicon has not been clear about the process of bilingual categorization. This study proposed to carefully categorize bilingual participants, as either balanced or unbalanced, using the Language Experience and Proficiency Questionnaire (LEAP-Q). This study also aimed at building on a particular study (Frenck-Mestre & Vaid, 1993) that examined confusion effects in English-French bilinguals. In that study, confusion effects were only found with respect to L1. However, with correct categorization of English-Spanish and Spanish-English bilinguals, accomplished using the LEAP-Q, this study proposed to examine confusion effects as they related to bilinguals who were both balanced and unbalanced in L2.

A final aim of the current study was to use well-known mathematical tasks (digit naming, arithmetic production, and arithmetic verification), to examine the Encoding Complex Model and see if that model could be extended or generalized from Chinese-English bilinguals to both English-Spanish and Spanish-English bilinguals. The Encoding Complex model is currently the only numerical processing model that has been extended to bilinguals, specifically Chinese-English bilinguals. It would be important for both the math cognition literature as well as the bilingual literature to determine if the Encoding Complex model could be directly extended to other types of bilinguals or if the model could be adapted in some way to accommodate other types of bilinguals. Also, it was important to investigate how or if the model could incorporate bilinguals who were not fluent in their second language because bilingual categorization is not currently discussed with regard to the Encoding-Complex model of bilingual numerical processing.

CHAPTER 3

METHODS

Participants

Bilingual participants, specifically English-Spanish and Spanish-English were recruited from the UNLV subject pool. Previous bilingual studies have used anywhere from 8 to 48 participants, with an average around 20. Eighty students participated in this study for course credit; 37 of the participants were English-Spanish bilinguals and 43 of the participants were Spanish-English bilinguals.

Materials

Demographic information was collected from all participants using a computer-based survey. Basic demographic information such as age, ethnicity, and year in school was obtained, and there was also information collected that was specific to this experiment. That information included the language in which participants learned math as well as in which language their high school math courses were taught. Math language was considered of particular importance based on previous research examining preferred langage effects (Marsh & Maki, 1976; McClain & Huang, 1982).

Language Experience and Proficiency Questionnaire (LEAP-Q): Areas of interest on the LEAP-Q were language competence, language acquisition, and prior and current language exposure. The questionnaire encompassed all of the categories recommended by Altarriba and Basnight-Brown (2007), including biographical data, language history data, and language proficiency data, and took an average of 10 minutes to complete for two languages. The LEAP-Q was used to decipher type of bilingual (English-Spanish or

Spanish-English) based on the order of language acquisition as well as to compute a measure of bilingual categorization (balanced vs. unbalanced in the second language). Participants self-scored themselves on a scale from 1 to 10 for how proficient they considered themselves to be when reading, speaking, and understanding whichever they considered to be their second language. An average of these scores was taken, with those scoring 5 or below categorized as unbalanced and those scoring between 5 and 10 categorized as balanced. The LEAP-Q is provided in Appendix 5.

Experimental Tasks and Stimuli

Digit Naming

The numbers 2 through 25, in Arabic digit form, were displayed on the computer screen one at a time. Participants were prompted before each trial in which language to name the digit. The prompt appeared in the middle of the screen for 1500 ms with the two words arranged vertically. If the participant was to answer in English, the prompt read "English, ingles" or "ingles, English," and if the participant was to answer in Spanish, the prompt read "Spanish, español" or "español, Spanish." The prompts were selected randomly. After a 500 ms blank screen, a randomly selected digit was displayed in the center of the screen until the participant produced the answer or until 5000 ms had been reached. If the 5000 ms time limit was reached, a display appeared prompting the entry of the digit as well as the language in which the digit was said. Trials that exceeded the 5000 ms time limit were given special codes and RTs for those trials were not included in the final analyses. Participants were given one practice block of 10 trials and two experimental blocks of 24 trials each. Each digit was named once in each language condition for a total of 48 trials.

Addition Production

Single digit addition problems were presented to participants horizontally in the center of the screen in the format $a + b$. The stimuli were constructed from the 56 possible nontie, pairwise combinations of the integers 2-9, and presentation format was always in digit form. One and zero were not used as addends because it is generally conceded in the literature that participants tend to use rules instead of direct retrieval for problems involving one and zero addends. With regard to the within subjects variable problem size, addition problems were categorized as small if the sum was smaller than ten and large if the sum was equal to or larger than ten, following the methods of Ashcraft and Stazyk (1981). The task required participants to verbally produce the answer in the language specified by a prompt before each trial. Participants were given one block of 10 practice trials and two blocks of 28 experimental trials; during the experimental trials, participants answered each problem once in English and once in Spanish. Answer format alternated across trials such that, prior to each trial, a language prompt was shown on the screen. The language prompts followed the same format as those for the digit naming task described above with a duration of 1500 ms, and the prompts were selected randomly so that participants could not predict whether they would need to produce their answer in English or Spanish until right before each trial. There was a 500 ms blank screen in between the language prompt and the addition problem. Once the addition problem was on the screen, participants had 5000 ms in which to respond. If participants did not respond in the 5000 ms, the trial was coded accordingly and the RT for that trial was excluded from data analysis.

Multiplication Production

Single digit multiplication problems were presented to participants horizontally in the center of the screen in the format *a* x *b*. The stimuli were constructed from the 56 possible nontie, pairwise combinations of the integers 2-9, and presentation format was always in digit form. One and zero were not used as multiplicands for the same reasons mentioned above for the addition production task. Multiplication problems were categorized as small if the product was less than 20 and large if the product was between 21 and 81, following the methods of Allen, Ashcraft and Weber (1992).The task required participants to verbally produce an answer in the language specified by a prompt before each trial. Participants were given one block of 10 practice trials and two blocks of 28 experimental trials; during the experimental trials, participants answered each problem once in English and once in Spanish. Answer format alternated randomly across trials such that, prior to each trial, a language prompt, following the same format as in the digit naming task, was shown on the screen for 1500 ms. Participants could not predict whether they were going to have to produce the answer to the multiplication problem in English or Spanish. A 500 ms blank screen was presented between the language prompt and the multiplication problem. The problem appeared on the screen until the participant answered or until 5000 ms had gone by. If the 5000 ms limit was reached, the trial was coded accordingly and the RT for that trial was excluded from data analysis.

Confusion Verification

Three blocks of 56 arithmetic trials, containing both simple addition problems of the form *a* + *b* = *c* and simple multiplication problems of the form *a* x *b* = *c*, were presented to participants for true/false verification. Once the stimuli were presented on the screen, participants either depressed the left mouse button if they verified the answer as true, or they depressed the right mouse buttin if they verified the answer as false. Trials were defined by the type of answer presented with the given problem: for true trials, c was the true answer to the problem, for associatively related trials, c was the sum of the problem for a multiplication trial and c was the product of the problem for an addition problem, and for neutral trials, c was mathematically unrelated to the correct answer to the problem, and the answer presented differed from the associatively related answer by ±1 or ±2. Presentation format alternated across blocks among Arabic Digit, English number word, and Spanish number word (e.g. 5 + 4 = 9, five + four = nine, and cinco + cuatro = nueve, respectively).

Procedure and Statistical Analyses

Upon arrival to the laboratory, participants completed and signed an informed consent sheet. They then used the computer for all other portions of the experiment, and an experimenter was also present. Participants completed the LEAP-Q first. This was followed by the digit naming, simple addition, simple multiplication, and confusion verification tasks presented in a counterbalanced order across participants. Instructions were both presented on the screen and read to the participant by the experimenter before each task. The experimenter explained each task as well as how to use the microphone, keyboard and mouse, which were provided for the experiment.

For the digit naming task, participants were told that a digit was going to appear on the screen. Prior to seeing the digit, they were going to see a prompt that told them in which language to name the digit. Participants were told to verbally produce responses into the microphone as quickly as they could, after which the experimenter would type in the digit that the participants said as well as the language in which they said it. The participants were told that they would be given several practice trials to get used to the microphone and that they would be told before the experimental trials began.

For both the addition and multiplication production tasks, participants were told that they would be seeing addition and multiplication problems on the screen. Before each problem appeared, they would see a prompt instructing them in which language to produce the answer. They were instructed to verbally produce their answers as quickly as they could into the microphone, and that they would be given several practice trials to get used to the microphone and the format of the task. Participants completed one full task consisting of only addition problems and one full task consisting of only multiplication problems.

For the confusion verification task, participants were instructed that they would be seeing either addition or multiplication problems presented on the screen, with answers. They were instructed to read the problem and determine if the answer presented with the problem was true or false. If the answer was true, they were told to press the left mouse button, and if the answer was false, they were told to press the right mouse button. No verbal answers were given during the confusion verification task. Participants received three counterbalanced blocks of trials varying in presentation format (digits, ENW, SNW).

After all of the math tasks were completed, participants completed the demographics survey. It was important that they be given the demographic survey at the end because the survey contained questions concering the language in which the participants learned elementary as well as high school math; if they received the demographics at the beginning, they may have tried to guess the purpose of the experiment and altered their peformance on the math tasks. After the demographic survey was completed, participants were de-briefed and asked if they had any questions regarding the experiment. The statistical analyses were slightly different for each of the mathematical tasks and will be discussed individually for each task in the next chapter. Reaction times and error rates were analyzed for the number naming, simple addition, simple multiplication, and confusion tasks.

CHAPTER 4

DATA ANALYSIS AND RESULTS

Before discussing the specifics of the data analysis and results, it is important to address some sampling issues that occurred during the data collection process. Ideally, bilingual participants would have fallen into one of four categories: English-Spanish (balanced and unbalanced) and Spanish-English (balanced and unbalanced). As the data were collected, it became clear that participants were only falling into three of the categories, with no participants categorized as Spanish-English unbalanced bilinguals. After 80 participants ran through the study, the data still did not have one participant categorized as Spanish-English unbalanced.

In order to be categorized as Spanish-English unbalanced, participants would have a second language of English and they would not consider themselves fluent in English. At UNLV, where almost all coursework is taught in English, finding Spanish-English participants who considered themselves to be unbalanced in English proved to be impossible, at least in the departmental subject pool. Going outside of the university setting to try to recruit participants would have risked the homogeneity of the sample in terms of demographic variables such as age and education level. That being the case, all analyses were conducted on only three categories of participants: English-Spanish (balanced and unbalanced) and Spanish-English (balanced only). After a preliminary analysis was conducted on all three groups of participants; two additional analyses were conducted: an ANOVA using both E-S and S-E balanced bilinguals to investigate type of bilingual and ANOVA using only E-S bilinguals to examine any effects of categorization. Because two ANOVAs were conducted using the same group of data (balanced E-S bilinguals), a Bonferroni adjustment was made to combat inflated Type I error. Therefore, all significance was compared to $p = .025$ instead of $p = .05$.

Eighty participants were run through the experiment. For the final analyses, the participant breakdown was as follows: LEAP-Q (n = 80), demographics (n = 78), digit naming (n = 65), addition production (n =64), multiplication production (n = 49), and confusion verification (n = 75). For a further breakdown of participants, refer to Table 1 in Appendix 4. Due to a software error, RT data were not collected for 15 participants. The fewest number of participants (n = 49) was analyzed for the multiplication production task; this was still more than twice the number of bilingual participants that have been recruited for previous bilingual studies. Other details about the number of participants the experimental tasks will be given with the results from each task.

Demographics and the LEAP-Q

Seventy-nine undergraduate students and one graduate student (age range: 18-40, with a mean of 20.31) consented to participate in the experiment. Due to a software failure in which the experiment froze, two participants were not able to complete their demographic information on the computer. Demographic data for those two participants did not get collected and were not included in the analyses. Means for several demographic variables can be found in Table 2 of Appendix 4. Due to the bilingual nature of the study, the sample was heavily ethnically skewed, with fifty-five of the seventy-eight participants reporting a Hispanic/Latino ethnicity. With regard to the language in which participants learned mathematics, sixty-three reported learning math in English and fifteen reported learning math in Spanish. Of the fifteen participants who reported learning math in Spanish, only three of them reported learning high school math in Spanish.

All eighty participants completed the LEAP-Q, and the between subjects variables, type of bilingual and bilingual categorization, were obtained using information from that questionnaire. Type of bilingual was determined based on language acquisition. If the participant learned English first, then they were typed English-Spanish; if they learned Spanish first, they were typed Spanish-English. If participants reported learning both languages at the same time, they were asked which one they mostly spoke at home, and that was used to determine their type. A bilingual categorization score (balanced vs. unbalanced), which was a measure of fluency in the second language, was obtained by taking participants' self-reported scores for how well they spoke, read, and understood their second language. Each of the three scores was out of 10 points, and the three scores were averaged to obtain a categorization. Participants who had an average rating of 5 or below were categorized as unbalanced and those with an average above 5 were categorized as balanced in the second language.

Digit Naming Task (Reaction Time Data)

For the digit naming task, within subjects factors included digit (2-25) and response language (Spanish or English) and the between subjects factor was type of bilingual (English-Spanish vs. Spanish-English). Because of a software error, reaction times were not collected for 15 participants; therefore, reaction times for only 65 participants were included in the analyses. Only reaction times for correct trials were analyzed; if the participant incorrectly produced the digit, produced the correct digit in the wrong language, or both, the reaction time was replaced with the mean for that group. Microphone errors were also excluded and replaced with the group mean. Outliers were defined as those reaction times falling above or below two and a half standard deviations of the group mean. For the digit naming task, out of 3120 individual trials, 18 digit errors, 99 language errors, 45 microphone errors, 17 time-outs (participant did not give an answer within 5000 ms), and 76 outliers were removed and replaced with the group mean.

Preliminary Full Sample Analyses

An initial analysis was run on the entire sample of 65 participants with reaction time data using a 2(type of bilingual) x 2(response language) x 24(digit) repeated measures ANOVA. There were significant main effects of response language ($F(1, 63) = 74.802$, $p = .000$, $\eta_p^2 = .543$) and digit ($F(23, 1449) = 3.842$, $p = .000$, $\eta_p^2 = .057$). Overall, naming digits in Spanish took longer than naming digits in English (mean RTs = 842 and 686 ms, respectively). Initial results also indicated a response language x type of bilingual interaction ($F(1, 63) = 28.457$, $p = .000$, $\eta_p^2 = .311$). According to the results, the E-S bilinguals were significantly slower when naming digits in Spanish than the S-E bilinguals (mean RTs = 925 and 759 ms, respectively), which makes sense considering Spanish was the second language for the E-S bilinguals. However, this interaction was suspected to be partially due to the fact that both balanced and unbalanced bilinguals were in the E-S sample while only balanced bilinguals were in the S-E sample. Other significant interactions included a digit x type of bilingual interaction ($F(23, 1149) = 2.626$, $p = .000$, $\eta_p^2 = .040$), a response language x digit interaction ($F(23, 1449) = 4.513$, $p = .000$, $\eta_p^2 = .067$), and a response language x digit x type of bilingual interaction

(F(23, 1449) = 2.284, p = .001, η_p^2 = .035). To maintain clarity, only the response language x digit interaction is presented (Appendix 4, Figure 1). The interaction shows the same pattern as the others in that group: digits were slower to be named in Spanish than in English. The three-way interaction indicated that, although both E-S and S-E bilinguals were equally fast to name digits in English, E-S bilinguals were slower than S-E bilinguals to name digits in Spanish.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

Another 2 (type of bilingual) x 2 (response language) x 24 (digit) ANOVA was run on only the balanced bilingual participants (both E-S and S-E, n = 56). Once again, the between subject variable was type of bilingual (E-S vs. S-E) and the two within subjects variables were response language (English vs. Spanish) and digit (2-25). The analysis for the balanced-only sample also produced significant main effects for response language (F(1, 54) = 59.999, p = .000, η_p^2 = .526) and digit (F(23, 1242) = 3.694, p = .000, η_p^2 = .064); once again, digits were slower to name in Spanish than in English (mean RTs = 794 and 688 ms, respectively). However, with only the balanced bilinguals included in the analyses, the between subjects variable type of bilingual was no longer significant with regard to naming digits in English and Spanish (F(1, 54) = .498, p = .483, η_p^2 = .009). With both groups reporting fluency in the second language, it made sense that there were no significant reaction time differences between the two groups of bilinguals when naming digits in English or in Spanish. All of the interactions from the main analysis were still significant for the analysis containing only balanced bilinguals. There were significant interactions for response language x type of bilingual (F(1, 54) = 11.609, p = .001, η_p^2 = .177) , digit x type of bilingual (F(23, 1242) = 2.288, p = .001, η_p^2 =

.041), response language x digit (F(23, 1242) = 4.075, p = .000, η_p^2 = .070), and response language x digit x type of bilingual (F(23, 1242) = 1.978, p = .004, η_p^2 = .035). The three-way interaction is displayed in Appendix 4 (Figures 2a & 2b). Overall, pairwise comparisons indicated that digits were significantly slower to be named in Spanish than in English (p = .000) for both E-S and S-E bilinguals; however, there were no significant differences in reaction time between E-S and S-E answering in English (p = .454) or answering in Spanish (p = .087).

E-S Bilinguals Only (Investigating Categorization)

To examine categorization (second language fluency), a separate analysis was conducted using only E-S bilinguals (n = 28), because the sample was missing a S-E unbalanced group. A 2 (categorization) x 2 (response language) x 24 (digit) ANOVA was conducted on the E-S bilinguals; the only change was that the between subjects variable from the previous analyses (type of bilingual) was now categorization (balanced vs. unbalanced). That analysis also revealed significant main effects of response language (F(1, 26) = 100.379, p = .000, η_p^2 = .794) and digit (F(23, 598) = 4.034, p = .000, η_p^2 = .134) as well as categorization (F(1, 26) = 8.267, p = .008, η_p^2 = .241). Once again, digits were faster to be named in English than in Spanish (mean RTs = 671 and 978 ms, respectively), and, including both languages, unbalanced E-S bilinguals were significantly slower to name digits than balanced E-S bilinguals (mean RTs = 896 and 753 ms, respectively). Significant interactions included response language x categorization (F(1, 26) = 25.034, p = .000, η_p^2 = .491), digit x categorization (F(23, 598) = 2.708, p = .000, η_p^2 = .094), response language x digit (F(23, 598) = 3.821. p = .000, η_p^2 = .128), and response language x digit x categorization (F(23, 598) = 1.849, p =

.010, η_p^2 = .066). As expected, both balanced and unbalanced E-S bilinguals performed similarly when naming digits in English (pairwise comparisons yielded p = .801); however, the unbalanced E-S bilinguals had significantly higher reaction times when naming digits in Spanish than the balanced E-S bilinguals (pairwise comparisons yielded p = .000) The interaction is displayed as Figure 3 in Appendix 4. The three-way interaction can be seen in Figures 4a and 4b of Appendix 4, and, as expected, the unbalanced E-S bilinguals were equally as fast as the balanced E-S bilinguals at naming digits in English but significantly slower to name digits in Spanish than the balanced E-S bilinguals.

Digit Naming Task (Error Rates)

The digit naming task gave participants the possibility of making three different types of errors: naming the incorrect digit, naming the correct digit in the wrong language, or both naming the incorrect digit in the wrong language. Therefore, it was possible to complete two different analyses for the error rates, one for digit errors and one for language errors; however, naming digits was such a simple task, which resulted in ceiling effects. Only 18 out of 3120 individual trials, .005% of the data, contained errors in which the participant named the incorrect digit; therefore, an error rate analysis of digit errors would not have added any additional information to the results. As far as language errors were concerned, 99 of the trials, 3% of the data, contained errors where the participant responded in the incorrect language. Once again, an ANOVA would not have produced any significant differences with so few of the overall trials containing errors; however it was interesting to break down the language errors to investigate which bilinguals were making the language errors as well as when they tended to make the errors. Of the language errors, 37 were digits supposed to be named in English that were incorrectly named in Spanish, and 62 were digits that were supposed to be named in Spanish that were incorrectly named in English. Also, 62 of the language errors were made on trials that involved a language switch (on the previous trial, the digit was supposed to be named in the other language) and 37 of the language errors were made on no-switch trials. Finally, 40 of the 49 participants who made language errors were balanced bilinguals and considered themselves relatively fluent in both languages. According to the data, being fluent in both languages created more language interference when naming digits.

Addition Production Task (Reaction Time Data)

The reaction time data for the addition production task fell into four categories: small problems answered in English, small problems answered in Spanish, large problems answered in English, and large problems answered in Spanish. For each participant, reaction time data was averaged for each type of answer. A 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was performed on the aggregate data. Only reaction times for correct trials were analyzed; if the participant produced the wrong answer, produced the correct answer in the wrong language, or both, the reaction time was replaced with the mean for that group. Microphone errors were also excluded and replaced with the group mean. Outliers were defined as those reaction times falling above or below two and a half standard deviations of the group mean. For the addition production task, out of 4224 individual trials, 158 math errors, 195 language errors, 26 incorrect answer and incorrect language errors, 167 microphone errors, 75 time outs (the participant did not answer the problem within 5000 ms), and 43 outliers were removed and replaced with the group mean. Group means were determined per participant. There were twelve small problems answered in English and Spanish and sixteen large problems answered in English and Spanish, creating four groups. If a participant was missing reaction times in a group, that reaction time was replaced with the mean for that particular participant for that individual group.

Preliminary Full Sample Analyses

Due to a computer malfunction, the addition production task froze in the middle of the trial block for one participant; therefore, initial analyses were run on 63 participants. The ANOVA for the full sample produced significant main effects of problem size ($F(1, 62) = 170.405$, $p = .000$, $\eta_p^2 = .733$) and response language ($F(1, 62) = 36.727$, $p = .000$, $\eta_p^2 = .372$). Traditional problem size effects were found such that large problems took significantly longer to solve than small problems (mean RTs = 1800 and 1300 ms, respectively). Also, participants took significantly longer to answer in Spanish than in English (mean RTs = 1622 and 1479 ms, respectively). There was also a significant response language x type interaction ($F(1, 62) = 19.606$, $p = .000$, $\eta_p^2 = .240$). The preliminary full sample analysis indicated that S-E bilinguals were more consistent in their response times and much slower than E-S bilinguals when answering in English. Also, E-S bilinguals were significantly slower answering in Spanish than in English however, this sample included both balanced and unbalanced E-S bilinguals.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

To better examine the relationship between response language and type of bilingual, a separate 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was run on only the balanced bilingual participants (n = 54). Results indicated significant main effects of problem size ($F(1, 53) = 122.206$, $p = .000$, $\eta_p^2 = .697$) and response language ($F(1, 53) = 20.390$, $p = .000$, $\eta_p^2 = .278$) as well as a significant response language x type of bilingual interaction ($F(1, 53) = 9.943$, $p = .004$, $\eta_p^2 = .144$). Once again, a significant problem size effect was obtained such that large problems took significantly longer to solve than small problems (mean RTs = 1783 and 1288, respectively). Also, on the whole, answering in Spanish took significantly longer than answering in English (mean RTs = 1592 and 1478, respectively). In reference to the significant interaction, S-E bilinguals were slower to answer addition problems overall than E-S bilinguals, but they were consistent in their reaction times whether answering in English or in Spanish whereas the E-S bilinguals, who considered themselves fluent in Spanish, took significantly longer to answer addition problems in Spanish than in English (Figure 5).

E-S Bilinguals Only (Investigating Categorization)

Finally, to assess the between subjects variable categorization (second language fluency), a third 2 (categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on only the E-S bilinguals (n = 27), because all of the S-E bilinguals were balanced in both languages. Main effects were found for problem size ($F(1, 26) = 70.820$, $p = .000$, $\eta_p^2 = .731$) and response language ($F(1, 26) = 47.338$, $p = .000$, $\eta_p^2 = .645$). In concordance with the previous analyses in this section, large problems took significantly longer to solve than small problems (mean RTs = 1760 and 1263 ms, respectively) and answering in Spanish took significantly longer than answering in English (mean RTs = 1651 and 1372 ms, respectively). Application of a Bonferroni correction left the analysis without any significant interactions; however the response language x categorization interaction approached significance ($F(1, 26) = 4.911$, $p = .036$, $\eta_p^2 = .159$) and was worth mentioning in the results. The interaction (Figure 6) produced exactly what would be expected: E-S bilinguals all took the same amount of time to produce answers to addition problems in English, but there was a marginally significant difference between balanced and unbalanced E-S bilinguals when answering addition problems in Spanish.

Addition Production Task (Math Errors)

The data were aggregated by problem size and response language and a 2 (type of bilingual/categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the average math error rates for each subject on each type of problem. No main effects of response language were found with regard to math errors in any of the analyses: the full sample, the balanced only sample, or the E-S only sample. There were main effects of problem size in all three analyses: full sample ($F(1, 62) = 30.615$, $p = .000$, $\eta_p^2 = .331$), E-S only sample ($F(1, 26) = 7.498$, $p = .011$, $\eta_p^2 = .224$), balanced only ($F(1, 53) = 23.257$, $p = .000$, $\eta_p^2 = .305$). As expected, significantly fewer math errors were made for small problems than for large problems (mean error rates = 2.4% and 8.6%, 2.5% and 8.5%, and <1% and 5.7% for the full sample, balanced sample, and E-S only sample, respectively).

Addition Production Task (Language Errors)

Preliminary Full Sample Analyses

The data were aggregated by problem size and response language and a 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the average language error rates for each subject on each type of problem. For the full sample (n = 63), significant main effects were found for problem size (F(1, 62) = 10.738, p = .002, η_p^2 = .148), response language (F(1, 62) = 13.727, p = .000, η_p^2 = .181), and type of bilingual (F(1, 62) = 5.812, p = .019, η_p^2 = .086). More language errors were made when producing answers to large problems than small problems (mean error rates = 8.1% and 5.1% , respectively), when producing answers in Spanish compared to English (mean error rates = 8.3% and 5.0%, respectively), and S-E bilinguals made significantly more language errors than E-S bilinguals (mean error rates = 8.3% and 5.0%, respectively). Once again, it was necessary to examine the balanced-only group of bilinguals to see if any of the above effects occurred because the E-S sample contained both balanced and unbalanced bilinguals while the S-E sample contained only balanced bilinguals.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

A second 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the aggregate language error rate data for the balanced only sample of bilinguals (n = 54). As expected, the significant main effect of type of bilingual found for the full sample did not reach significance for the balanced only sample (F(1, 53) = 3.328, p = .074, η_p^2 = .059). Significant main effects were found for both problem size (F(1, 53) = 7.978, p = .007, η_p^2 = .131) and response language (F(1,

53) = 8.212, p = .006, η_p^2 = .134). Similarly to the full sample, more language errors were made for large problems than for small problems (mean error rates = 8.3% and 5.3%, respectively) and more language errors were made when participants responded in Spanish than in English (mean error rates = 8.2% and 5.5%, respectively).

E-S Bilinguals Only (Investigating Categorization)

A final analysis was conducted on only the E-S bilinguals (n = 27) to investigate whether there would be any effects of categorization (second language fluency) on the number of language errors. The analysis was a 2 (categorization) x 2 (problem size) x 2 (response language) mixed ANOVA on the aggregate language error rate data; a significant main effect was found for response language (F(1, 26) = 9.512 p = .005, η_p^2 = .268), and a marginally significant main effect was found for problem size (F(1, 26) = 5.201 p = .031, η_p^2 = .167. This final analysis showed more language errors when participants were asked to respond in Spanish than in English (mean error rates = 7.0% and 2.5%, respectively) and more language errors to large problems than to small problems (mean error rates = 6.2% and 3.4%, respectively). Overall, more language errors were made on large addition problems than on small addition problems; this was the case regardless of the type of bilingual or categorization. Also, more language errors were made when bilinguals were producing the answer in Spanish than when they were producing the answer in English, also regardless of the type of bilingual and categorization.

As with the digit naming data, a breakdown was done to examine language errors. There were 195 language errors that occurred on switch trials (the trial before required participants to respond in the other language) and 95 language errors that occurred on no-switch trials (the response language had not changed from the previous trial). For the language errors that were made during switch trials, 110 were errors in which participants incorrectly answered in English and 85 were errors in which participants incorrectly answered in Spanish. Of the 59 bilinguals that made at least one language error during the course of the addition production task, 53 were balanced and 6 were unbalanced. Once again, the majority of the language errors were made by balanced bilinguals, suggesting more language interference as fluency in the second language increases.

Multiplication Production (Reaction Time Data)

Before discussing the reaction time data for the multiplication task, it is important to talk about the difficult nature of the task. Firstly, there were several different types of errors that could be made by the participant that would result in a reaction time needing to be replaced (i.e., a math error, a language error, or both). Secondly, participants were shown the language prompt, which then disappeared and was replaced with the multiplication problem. Participants then needed to retrieve the answer, remember the language in which they were supposed to respond, and then possibly translate the answer into the correct language, depending on whether the answer was required in their first or second language as well as whether they were balanced or unbalanced in the second language. Errors on the multiplication task were extensive, with some participants making errors on more than fifty percent of the trials. There were four cells containing data for the multiplication task: 12 small problems answered in English, 12 small problems answered in Spanish, 16 large problems answered in English, and 16 large problems answered in Spanish. For the participants that made more than fifty percent errors, entire cells consisted of only errors of one type or another, making it impossible to replace the data, because there was no data from which to calculate a group mean. Because of the strict exclusion that was being done with regard to errors, a criterion needed to be established to include as many participants as possible without sacrificing the integrity of the data. After examining several cutoffs, a thirty percent cutoff was established; participants with more than thirty percent errors were excluded from analysis because there would be too much of the data missing and in need of replacement. Therefore, fifteen participants (participants 16, 20, 22, 24, 25, 33, 34, 47, 50, 51, 60, 63, 71, 72, 73) were excluded from the reaction time analysis for the multiplication production task.

Reaction time data for 49 participants fell into four categories: small problems answered in English, small problems answered in Spanish, large problems answered in English, and large problems answered in Spanish. For each participant, reaction time data was averaged for each type of answer. A 2 (type of bilingual/categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was performed on the aggregate data. Only reaction times for correct trials were analyzed; if the participant incorrectly produced the answer, produced the correct answer in the wrong language, or both, the reaction time was replaced with the mean for that group. Microphone errors were also excluded and replaced with the group mean. Outliers were defined as those reaction times falling above or below two and a half standard deviations of the group mean. For the multiplication production task, out of 2744 individual trials, 167 incorrect answer errors, 130 language errors, 14 incorrect answer and incorrect language errors, 154 microphone errors, 92 time outs (the participant did not answer the problem within 5000 ms), and 40 outliers were removed and replaced with group means. Group means were established according to the same criteria mentioned above for the addition production task.

Preliminary Full Sample Analyses

For the full sample (n = 49), main effects were found for both problem size ($F(1, 47) = 217.777$, $p = .000$, $\eta_p^2 = .822$) and response language ($F(1, 47) = 17.615$, $p = .000$, $\eta_p^2 = .273$). The traditional problem size effect was found with large problems taking significantly longer to solve than small problems (mean RTs = 1834 and 1306 ms, respectively. Also, all problems were significantly slower to be answered in Spanish than in English (mean RTs = 1645 and 1495 ms, respectively. There was also a significant response language x type of bilingual interaction ($F(1, 47) = 22.337$, $p = .000$, $\eta_p^2 = .322$) For the full sample, both types of bilinguals were equally fast at producing multiplication answers in English; however, the E-S bilinguals were significantly slower at producing multiplication answers in Spanish. This was not surprising considering that the full sample contained both balanced and unbalanced E-S bilinguals but only balanced S-E bilinguals. It would be more interesting to see if the interaction was still significant when examining only the balanced bilinguals in the sample.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

Analyses of the balanced bilinguals (n = 41) who participated in the study produced main effects of both problem size ($F(1, 40) = 168.950$, $p = .000$, $\eta_p^2 = .809$) and response language ($F(1, 40) = 9.917$, $p = .003$, $\eta_p^2 = .199$) as well as a significant response language x type interaction ($F(1, 47) = 14.257$, $p = .001$, $\eta_p^2 = .263$). Once again there was a traditional problem size effect, with large problems taking significantly longer to solve than small problems (mean RTs = 1798 and 1270 ms, respectively). Also, production responses were slower in Spanish than in English (mean RTs = 1581 and 1486 ms, respectively). Finally, even with only balanced bilinguals in the sample, pairwise comparisons showed E-S bilinguals answering significantly slower in Spanish than in English (p = .000); however, there were no significant differences between E-S and S-E bilinguals when answering in English (p = .942) or Spanish (p = .048). The interaction is displayed as Figure 7 in Appendix 4.

E-S Bilinguals Only (Investigating Categorization)

Finally, a 2 (categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on only the E-S bilinguals (n = 23). That analysis produced main effects of problem size ($F(1, 22) = 116.132$, $p = .000$, $\eta_p^2 = .841$) and response language ($F(1, 22) = 43.786$, $p = .000$, $\eta_p^2 = .666$) and two significant interactions: response language x categorization ($F(1, 47) = 9.841$, $p = .005$, $\eta_p^2 = .309$) and problem size x response language ($F(1, 47) = 9.597$, $p = .005$, $\eta_p^2 = .304$). Similar findings for the main effects found in the full and balanced-only samples above were found with the E-S only sample. Large problems were significantly slower to be answered than small problems (mean RTs = 1998 and 1424 ms, respectively) and multiplication answers took longer to produce in Spanish than in English (mean RTs = 1911 and 1512 ms, respectively). Figure 8 in Appendix 4 displays the response language x categorization interaction, and it can be seen that the unbalanced E-S bilinguals were significantly slower to answer problems in Spanish than the balanced E-S bilinguals (pairwise comparisons yielded p = .009). The problem size x response language interaction can be seen in Figure 9 of Appendix 4; although producing multiplication answers took longer overall when producing them in Spanish than in English, producing answers in Spanish also led to a greater problem size effect, regardless of categorization.

Multiplication Production (Math Errors)

Preliminary Full Sample Analyses

The data were aggregated by problem size and language and a 2 (type of bilingual or categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the average error rates for each subject on each type of problem. The between subjects variable was type of bilingual (E-S, S-E) and the within subjects variables were problem size (small, large) and response language (English, Spanish). For the full sample, a main effect was found for problem size ($F(1, 47) = 72.402$, $p = .000$, $\eta_p^2 = .606$) as well as a marginally significant response language x type of bilingual interaction ($F(1, 47) = 4.930$, $p = .031$, $\eta_p^2 = .095$). The traditional problem size effect was found with respect to error rates for the production of multiplication facts. Significantly more errors were made when producing the answers to large problems than when producing the answers to small problems (mean error rates = 13.2% and 1.9%, respectively). There was no main effect of language, and even though there was a marginally significant response language x type of bilingual interaction, this was most likely due to the fact that both balanced and unbalanced E-S bilinguals were included in the full sample analysis, but only balanced S-E bilinguals were included.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

A second 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on only the balanced bilinguals in the sample. The only significant finding for the balanced only sample was a main effect of problem size (F(1, 40) = 57.185, p = .000, η_p^2 = .588). Once again, more errors were made when producing answers to large problems than when producing answers to small problems (mean error rates = 12.8% and 1.9%, respectively), regardless of response language. The response language x type of bilingual interaction disappeared upon analysis of only balanced bilinguals (F(1, 40) = 2.784 p = .103, η_p^2 = .065).

E-S Bilinguals Only (Investigating Categorization)

A final 2 (categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the E-S bilinguals to assess categorization. Main effects were found for both problem size (F(1, 22) = 42.491, p = .000, η_p^2 = .659) and response language (F(1, 22) = 7.082, p = .014, η_p^2 = .244) as well as a marginally significant problem size x response language interaction (F(1, 22) = 5.554, p = .028, η_p^2 = .202). Bilinguals made more errors when producing answers to large problems than to small problems (mean error rates = 14.5% and 1.8%, respectively). Specifically, E-S bilinguals made more errors when producing the answer in Spanish than in English (mean error rates = 10% and 6.2%, respectively), regardless of categorization (balanced vs. unbalanced). Finally, with regard to the marginally significant problem size x response language interaction, more math errors were made on large problems when producing the answer in Spanish (Appendix 4, Figure 10), which makes sense considering the entire sample used for that analysis consisted of bilinguals with Spanish as their second language.

Multiplication Production (Language Errors)

Preliminary Full Sample Analyses

As with the math error data the language error data were aggregated by problem size and language and a 2 (type of bilingual or categorization) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted on the average error rates for each subject on each type of problem. Once again, the within subjects variables were problem size (small vs. large) and response language (English vs. Spanish). The full sample mixed ANOVA produced significant main effects of problem size ($F(1, 47) = 10.158$, $p = .003$, $\eta_p^2 = .178$) and response language ($F(1, 47) = 9.834$, $p = .003$, $\eta_p^2 = .173$). For the full sample, more language errors were made for large problems than for small problems (mean error rates = 8% and 4.5%, respectively), and more language errors were made when responses were in Spanish than when responses were in English (mean error rates = 8.1% and 4.4%, respectively).

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

A 2 (type of bilingual) x 2 (problem size) x 2 (response language) mixed ANOVA was conducted using only balanced bilinguals. Significant main effects were found for problem size ($F(1, 40) = 7.283$, $p = .010$, $\eta_p^2 = .154$) and response language ($F(1, 40) = 7.958$, $p = .007$, $\eta_p^2 = .166$). More language errors were made when producing answers to large problems than to small problems (mean error rates = 8.3% and 4.8%, respectively) and more language errors were made when responses were required in Spanish than in English (mean error rates = 8.4% and 4.5%, respectively).

E-S Bilinguals Only (Investigating Categorization)

A final 2 x 2 x 2 ANOVA examined only E-S bilinguals to look at categorization. Significant main effects were found for problem size (F(1, 22) = 12.102 p = .002, η_p^2 = .355) and response language (F(1, 22) = 6.648 p = .017, η_p^2 = .232). In accordance with the analyses above, more errors were made for large problems then for small problems (mean error rates = 7.7% and 2.4%, respectively), and higher error rates were exhibited when E-S bilinguals were producing the answer in Spanish than when they produced the answer in English (mean error rates = 7.7% and 2.4%, respectively).

A breakdown was done to examine language errors. There were 112 language errors that occurred on switch trials (the trial before required participants to respond in the other language) and 47 language errors that occurred on no-switch trials (the response language had not changed from the previous trial). For the language errors that were made during switch trials, 67 were errors in which participants incorrectly answered in English and 45 were errors in which participants incorrectly answered in Spanish. Of the 44 bilinguals that made at least one language error during the course of the addition production task, 39 were balanced and 5 were unbalanced. Once again, the majority of the language errors were made by balanced bilinguals, suggesting more language interference as fluency in the second language increases.

Confusion Verification (Reaction Time Data: True Probes)

Seventy five participants completed the confusion verification task. After examining the data, one participant made over 50% errors, creating entire cells of data that were missing for that participant. Due to the high error rate, that participant was removed from the analyses. Also, because the analyses consisted of multiple ANOVAs, a Bonferroni adjustment was made to combat inflated Type I error. Therefore, all significance was compared to $p = .025$ instead of $p = .05$.

Preliminary Full Sample Analyses

Reaction time analysis was conducted on the true probes for the full sample (n = 74) using a 2 (type of bilingual) x 2 (operation: addition vs. multiplication) x 3 [presentation format: digit, English number word (ENW), Spanish number word (SNW)] mixed ANOVA. Main effects were found for both format ($F(2, 144) = 269.003$, $p = .000$, $\eta_p^2 = .789$) and operation ($F(1, 72) = 17.020$, $p = .000$, $\eta_p^2 = .191$). Overall, digit presentation received the quickest reaction times, followed by ENW and SNW presentations. Mean RTs were 1408, 2996, and 3253 ms, respectively. Multiplication was also significantly slower than addition, with mean RTs of 2448 and 2657 msec. There were two significant interactions, a format x type of bilingual interaction ($F(2, 144) = 7.066$, $p = .001$, $\eta_p^2 = .089$) and a format x operation interaction ($F(2, 144) = 18.725$, $p = .000$, $\eta_p^2 = .206$). When examining the format x type of bilingual interaction (Appendix 4, Figure 11), it could be seen that E-S bilinguals exhibited a marked difference in reaction time at each format; they were fastest in the digit condition, slower in the ENW condition, and slowest in the SNW condition. This would be expected considering E-S bilinguals had Spanish as their L2; however, the S-E bilinguals did not display the same pattern when viewing the full sample. S-E bilinguals performed similarly when presented with digits and also had significant slowing when presented with number words; however they had similar reaction times whether viewing English or Spanish number words. Again, this makes sense considering all of the S-E bilinguals in the sample were balanced. In that case, fluency was expected in both L1 and L2. Reaction times typically go up when participants are presented with number words compared to digits because they are not as practiced performing arithmetic operations presented in number word form. However, the E-S sample contained both balanced and unbalanced bilinguals, which could have accounted for the significant slowdown from ENW presentation to SNW presentation.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

For the balanced only sample (n = 62), the same 2 (type of bilingual) x 2 (operation) x 3 (presentation format) mixed ANOVA as described for the full sample was performed. Once again, there were significant main effects of format ($F(2, 120) = 218.230$, $p = .000$, $\eta_p^2 = .784$) and operation ($F(1, 60) = 15.456$, $p = .000$, $\eta_p^2 = .205$) and a significant format x operation interaction ($F(2, 120) = 15.417$, $p = .000$, $\eta_p^2 = .204$); however, when examining only the balanced participants, the format x type of bilingual interaction from the full analysis was no longer significant ($F(2, 120) = 2.503$, $p = .086$, $\eta_p^2 = .040$), which was expected. With regard to the main effect of format, digit presentation was significantly faster than either ENW or SNW presentation (pairwise comparisons yielded $p = .000$); mean RTs were 1371, 3001, and 3145 ms, respectively. Balanced bilinguals (bilinguals fluent in both languages) showed similar reaction times to solve arithmetic problems presented in number word format in L1 or L2. Multiplication problems were slower to verify as true than addition problems with mean RTs of 2393 and 2618 msec. The operation x format interaction showed slower reaction times with number word presentation for both addition and multiplication; however, for multiplication, the reaction times for verifying true probes were significantly slower for SNW than for ENW. Pairwise comparisons showed reaction times between addition and multiplication were not significantly different for the digit or ENW formats (*p* = .041 and .035, respectively) but that reaction times were significantly slower to verify multiplication than addition in the SNW format (*p* =.000).

<u>E-S Bilinguals Only (Investigating Categorization)</u>

To investigate categorization (balanced vs. unbalanced) a 2 (categorization) x 2 (operation) x 3 (presentation format) mixed ANOVA was conducted on only the E-S bilinguals. Main effects were found for both format ($F(2, 70) = 165.355$, $p = .000$, $\eta_p^2 = .825$) and operation ($F(1, 35) = 11.182$, $p = .002$, $\eta_p^2 = .242$) along with significant format x categorization ($F(2, 70) = 4.618$, $p = .013$, $\eta_p^2 = .117$) and format x operation ($F(2, 70) = 13.202$, $p = .000$, $\eta_p^2 = .274$) interactions. Once again, mean RTs increased for verifying true probes depending on presentation format with the fastest reaction time for digit presentation, followed by ENW and then SNW presentations (mean RTs = 1378, 2759, and 3463 ms, respectively). Multiplication took significantly longer than addition to verify true probes with mean RTs of 2654 and 2413, respectively. The format x categorization interaction (Appendix 4, Figure 12) showed expected results. Both balanced and unbalanced E-S bilinguals verified true probes with similar reaction times for both digit and ENW presentation, and then there was a nice split between the balanced and unbalanced E-S bilinguals when verifying true probes presented as SNW, with unbalanced bilinguals significantly slower to verify the answers to true probes than balanced bilinguals (pairwise comparisons yielded *p* = .000).

Confusion Verification (Error Rate Data: True Probes)

Preliminary Full Sample Analyses

Preliminary error rate analyses were conducted on all three groups using a 2 (type of bilingual) x 2 (operation) x 3 (presentation format) mixed ANOVA. Significant main effects were found for format ($F(2, 144) = 26.052$, $p = .000$, $\eta_p^2 = .266$) and operation ($F(1, 72) = 48.538$, $p = .000$, $\eta_p^2 = .403$) as well as significant operation x type of bilingual ($F(1, 72) = 7.563$, $p = .008$, $\eta_p^2 = .095$) and format x operation ($F(2, 144) = 32.338$, $p = .000$, $\eta_p^2 = .310$) interactions. When examining all three groups, there were more errors made while verifying true probes when the problem was presented as an ENW than when the problem was presented as a digit, and even more errors were made when the problem was presented as a SNW compared to both digit and ENW presentations (mean error rates = 4.2%, 8.6%, and 13.1%, respectively). Errors were also more likely to be made when verifying true multiplication problems than true addition problems (mean error rates = 11.1% and 6.2%, respectively). The significant operation x type of bilingual interaction indicated not only an increase in error rates when verifying true multiplication problems compared to true addition problems for both types of bilinguals, but this effect was significantly more pronounced for the S-E bilinguals (Appendix 4, Figure 13). With that being the case, further analysis should still produce a significant operation x type of bilingual interaction because all of the S-E bilinguals were balanced. With regard to the format x operation interaction, errors verifying true probes were not significantly different for addition problems across presentation format; however, for true multiplication probes, error rates steadily increased from the digit presentation to the ENE presentation to the SNW presentation (Appendix 4, Figure 14).

It was possible that this effect was due to the fact that unbalanced E-S bilinguals were included in the full analysis, and it was expected that when only the balanced participants were analyzed, that the interaction may have been smaller or disappeared completely.

E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1

The 2 (type of bilingual) x 2 (operation) x 3 (presentation format) mixed ANOVA on only the balanced bilinguals in the study revealed significant main effects of format ($F(2, 120) = 21.876$, $p = .000$, $\eta_p^2 = .267$) and operation ($F(1, 60) = 31.687$, $p = .000$, $\eta_p^2 = .346$) as well as significant operation x type ($F(1, 60) = 7.859$, $p = .007$, $\eta_p^2 = .116$) and format x operation ($F(2, 120) = 21.411$, $p = .000$, $\eta_p^2 = .263$) interactions. Once again, error rates verifying true probes increased from digit to ENW to SNW presentation (mean error rates = 4.0%, 9.2%, and 13.4%, respectively); however, with only the balanced bilinguals included in the analyses, the difference in error rates between the number word presentations was not as pronounced as it was for the three group analysis, which was to be expected. There were more errors verifying true multiplication problems than true addition problems (mean error rates = 11.1% and 6.6%, respectively), and the operation x type of bilingual interaction showed balanced S-E bilinguals with significantly higher error rates than balanced E-S bilinguals when verifying true multiplication problems than true addition problems. The figure was very similar to the one presented for the three group analysis and therefore was not replicated for this section (refer to Appendix 4, Figure 13). Interestingly, the operation x format interaction for the balanced bilinguals also looked similar to the interaction for all three groups. Verifying true addition problems exhibited approximately the same level of errors regardless of presentation, but the error rates for verifying true multiplication problems steadily rose across presentation formats (digit, ENW, SNW), respectively.

E-S Bilinguals Only (Investigating Categorization)

To investigate whether categorization had error rate effects when participants were verifying true probes, a 2 (categorization) x 2 (operation) x 3 (presentation format) mixed ANOVA was conducted. Main effects were found for both format ($F(2, 70) = 17.578$, $p = .000$, $\eta_p^2 = .334$) and operation ($F(1, 35) = 14.898$, $p = .000$, $\eta_p^2 = .299$) as well as a significant format x operation interaction ($F(2, 70) = 14.711$, $p = .000$, $\eta_p^2 = .296$). Significantly more errors were made when verifying true problems presented as SNW than either ENW or digits, regardless of categorization (mean error rates = 12.8%, 6.5%, and 2.5%, respectively). Also, the difference in error rates between digit presentation and ENW presentation was not very large. It was most likely the case that E-S bilinguals were not practiced in reading and performing arithmetic in Spanish number words, especially since the E-S bilinguals had English as their first language and learned math in English. Even though they could count to 100 in Spanish, their calculation mechanisms worked in English, possibly forcing them to translate each number in the presented problem into English before performing any calculations on it. After they retrieved the answer in English, they then had to translate that answer back to Spanish before being able to verify if the answer presented with the problem was true or false. E-S bilinguals made significantly more errors when verifying true multiplication problems than true addition problems (mean error rates = 8.9% and 5.6%, respectively); this effect could be seen in the format x operation interaction in which, regardless of format, approximately the same number of errors was made when E-S bilinguals were verifying true addition problems but made increasingly more errors across presentation format (digit, ENW, SNW) when verifying true multiplication problems. The graph for that interaction looked almost exactly like the graph of the interaction for the three group analysis and therefore was not included (refer back to Appendix 4, Figure 14).

Confusion Verification (Reaction Time Data: False Probes)

Preliminary Full Sample Analyses

Two types of false probes were presented: neutral false probes in which the answer presented was within ±1-3 of the correct answer and confusion probes in which the answer presented was associatively related to the problem (e.g. 3 + 4 = 12 or 3 x 4 = 7). Probe type was analyzed as a within subjects variable in the analysis. A 2 (type of bilingual) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA was conducted. Significant main effects were found for format ($F(2, 144) = 268.614$, $p = .000$, $\eta_p^2 = .789$), operation ($F(1, 72) = 5.784$, $p = .019$, $\eta_p^2 = .074$), and type of probe ($F(1, 72) = 15.004$, $p = .000$, $\eta_p^2 = .172$) along with significant format x type of bilingual ($F(2, 144) = 8.052$, $p = .000$, $\eta_p^2 = .101$), format x operation ($F(2, 144) = 4.833$, $p = .009$, $\eta_p^2 = .063$), format x probe type ($F(2, 144) = 10.898$, $p = .000$, $\eta_p^2 = .131$), and format x operation x probe type ($F(2, 144) = 6.661$, $p = .002$, $\eta_p^2 = .085$) interactions. For false probes, reaction time increased steadily across presentation format (digit, ENW, SNW; mean RTs = 1438, 2551, and 3373 ms, respectively). False addition problems took longer to verify than false multiplication problems showing mean RTs of 2492 and 2416 ms, respectively, and confusion probes took longer to verify as false than neutral probes (mean RTs = 2513 and 2395 ms, respectively). Small differences were observed for the format x type of bilingual interaction such that the S-E bilinguals were slightly slower than the E-S bilinguals to verify false probes presented as ENW and slightly faster than E-S bilinguals to verify false problems presented as SNW. This made sense considering S-E bilinguals had Spanish as their L1 and English as their L2. The format x operation interaction was only due to a small increase in RT for verifying false multiplication problems presented as SNW compared to verifying false addition problems presented as SNW. The digit and ENW presentations showed consistent verification reaction times across operations. The operation x probe type interaction showed similar verification times for confusion probes across operations; however, neutral probes were found to have significantly faster reaction times for multiplication problems than for addition problems (Appendix 4, Figure 15). Finally, the format x operation x probe type interaction was only due to a slight decrease in RT for neutral multiplication probes presented in SNW compared to neutral addition probes presented in SNW. All other presentation formats across neutral and confusion probes exhibited consistent RT performance across arithmetic operations.

<u>E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1</u>

The balanced bilinguals were also analyzed using a 2 (type of bilingual) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA. Significant main effects were found for format ($F(2, 120) = 226.867$, $p = .000$, $\eta_p^2 = .791$), operation ($F(1, 60) = 5.920$, $p = .018$, $\eta_p^2 = .090$), and probe type ($F(1, 60) = 8.431$, $p = .005$, $\eta_p^2 = .123$). Reaction times increased consistently across presentation format (mean RTs = 1409, 2548, and 3269 ms, respectively), addition problems took longer to verify as false than multiplication problems (mean RTs =2452 and 2365 ms, respectively) , and confusion probes took significantly longer to verify as false than neutral probes (mean RTs = 2458 and 2359 ms, respectively). Significant interactions included format x operation (F(2, 120) = 4.473, p = .013, η_p^2 = .069) and format x probe type (F(2, 120) = 6.075, p = .003, η_p^2 = .092). Both interactions were due to slight changes in RT when presented with Spanish number words. For the format x operation interaction, RT verification for false probes slightly decreased from addition to multiplication, and for the format x probe type interaction, RT verification for false probes slightly increased from neutral probes to confusion probes. All other RTs were consistent across format, operation, and probe type.

E-S Bilinguals Only (Investigating Categorization)

To investigate any effects on reaction times involving categorization, a 2 (categorization) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA was conducted using only E-S bilinguals. Categorization did not reach significance for the RT analysis; however main effects were found for format (F(2, 70) = 170.700, p = .000, η_p^2 = .830) and probe type (F(1, 35) = 9.547, p = .004, η_p^2 = .214). For false probes, SNW presentation took longer to verify than either ENW or digit presentation (mean RTs = 3588, 2322, and 1372 ms, respectively), and confusion probes took significantly longer to verify as false than neutral probes (mean RTs = 2486 and 2368 ms, respectively). Significant interactions were found for format x probe type (F(2, 70) = 7.846, p = .001, η_p^2 = .183), operation x probe type (F(1, 35) = 6.673, p = .014, η_p^2 = .160), and format x operation x probe type (F(2, 70) = 9.796, p = .000, η_p^2 = .219). The format x probe type interaction was only significant due to a slight increase in RT when E-S bilinguals were verifying confusion probes presented in Spanish number words. The operation x probe type interaction showed similar RTs for both neutral and confusion probes across false addition problems; however, for multiplication problems, confusion probes took significantly longer to verify than neutral probes (Appendix 4, Figure 16). Finally, the format x operation x probe type interaction was due to a slight shift in RT for participants verifying neutral probes presented in Spanish number words. Participants were slightly faster to verify multiplication problems as false than addition problems.

Confusion Verification (Error Rate Data: False Probes)

Preliminary Full Sample Analyses

Error rates were analyzed for all three groups using a 2 (type of bilingual) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA. The analysis revealed main effects of format ($F(2, 144) = 294.191$, $p = .000$, $\eta_p^2 = .803$), operation ($F(1, 72) = 7.740$, $p = .007$, $\eta_p^2 = .097$), and probe type ($F(1, 72) = 46.695$, $p = .000$, $\eta_p^2 = .393$). Full sample analysis showed significantly more errors to verify false problems presented in SNW compared to problems presented in either ENW or digit form both of which showed approximately equal numbers of errors (mean error rates = 15.9%, 3.7%, and 3.6%, respectively). Also, more errors were made to confusion probes than to neutral probes (mean error rates = 9.2% and 6.2%, respectively). Interestingly, more errors were made when verifying false addition problems then when verifying false multiplication problems (mean error rates = 8.5% and 7.0%, respectively). Significant interactions included operation x type of bilingual ($F(1, 72) = 5.419$, $p = .023$, $\eta_p^2 = .070$), format x operation ($F(2, 144) = 6.461$, $p = .002$, $\eta_p^2 = .082$), format x probe type ($F(2, 144) =$

4.245, $p = .016$, $\eta_p^2 = .056$), operation x probe type ($F(1, 72) = 429.235$, $p = .000$, $\eta_p^2 = .856$), and format x operation x probe type ($F(2, 144) = 235.089$, $p = .000$, $\eta_p^2 = .766$). The operation x type interaction showed similar error rates for both E-S and S-E bilinguals when verifying false multiplication problems. Also E-S bilinguals exhibited similar error rates for both false addition and false multiplication problems; however, S-E bilinguals made significantly more errors when verifying false addition problems than when verifying false multiplication problems. The format x operation as well as the format x probe type interactions were very similar in that more errors were made across operation and across probe type to false problems that were presented as Spanish number words. The operation x probe type interaction yielded some interesting results; for all groups, more errors were made when verifying neutral probe multiplication problems than confusion probe multiplication problems. Also, significantly more errors were made to confusion probe addition problems than neutral probe addition problems. This was a very interesting result; however, since the three group analysis was preliminary, it would need to be seen again in subsequent analyses. Finally, the format x operation x probe type interaction showed that, for neutral probes, significantly more errors were made with multiplication problems presented as SNW; however, for confusion probes, significantly more errors were made to addition problems presented as SNW. This was also very interesting, but would require further analyses before any explanations could be considered.

<u>E-S and S-E Analysis: Only Balanced Bilinguals with Differing L1</u>

A 2 (type of bilingual) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA was conducted using only balanced bilinguals. Significant main effects were found for format ($F(2, 120) = 205.663$, $p = .000$, $\eta_p^2 = .774$), operation ($F(1, 60) = 3.623$, $p = .016$, $\eta_p^2 = .093$), and probe type ($F(1, 60) = 38.519$, $p = .000$, $\eta_p^2 = .391$). As in the three group analysis, significantly higher error rates were made when verifying false probes presented as SNW than when they were presented as either ENW or digits, which had similar error rates (mean error rates = 15.8%, 3.8%, and 3.9%, respectively). Also, more errors were made when verifying false addition compared to multiplication problems (mean error rates = 8.7% and 7.0%, respectively), and more errors were made to confusion probes than neutral probes (mean error rates = 9.4% and 6.3%, respectively). Significant interactions included format x operation ($F(2, 120) = 6.534$, $p = .002$, $\eta_p^2 = .098$), format x probe type ($F(2, 120) = 4.082$, $p = .019$, $\eta_p^2 = .064$), operation x probe type ($F(1, 60) = 334.627$, $p = .000$, $\eta_p^2 = .848$), and format x operation x probe type ($F(2, 120) = 165.386$, $p = .000$, $\eta_p^2 = .734$). Both the format x operation and format x probe type interactions did not provide a lot of information except that more errors were made when false problems were verified as Spanish number words. The operation x probe type interaction was once again very interesting (Appendix 4, Figure 17). Significantly more errors were made when verifying neutral probe multiplication problems and confusion probe addition problems, especially with regard to the confusion probe addition problems, which showed a nearly 15% error rate. Finally, the format x operation x probe type interaction showed that, for neutral probes, multiplication problems presented as SNW exhibited a 28.3% error rate. On the other hand, for confusion probes, addition problems presented as SNW exhibited a 28.7% error rate (Appendix 4, Figures 18a & 18b). Both of those interactions were very interesting and will be brought up again in the discussion.

E-S Bilinguals Only (Investigating Categorization)

Finally, to examine any effects of categorization, a 2 (categorization) x 2 (operation) x 2 (probe type) x 3 (presentation format) mixed ANOVA was conducted on only the E-S bilinguals. Significant main effects were found for format ($F(2, 70) = 215.635$, $p = .000$, $\eta_p^2 = .860$) and probe type ($F(1, 35) = 15.852$, $p = .000$, $\eta_p^2 = .312$). Unlike the previous two analyses, there was not a significant main effect of operation when looking at only E-S bilinguals. However, significantly more errors were made when the problems were presented as SNW compared to ENW or digits (mean error rates = 16.2%, 2.0%, and 2.5%, respectively), and more errors were made to confusion probes than to neutral probes (mean error rates = 8.1% and 5.7%, respectively). Significant interactions included operation x probe type ($F(1, 35) = 321.761$, $p = .000$, $\eta_p^2 = .902$) and format x operation x probe type ($F(2, 70) = 167.386$, $p = .000$, $\eta_p^2 = .827$). Similar to the balanced only group, the E-S bilinguals displayed higher error rates for neutral multiplication problems as well as confusion addition problems, specifically those presented as SNW.

CHAPTER 5

DISCUSSION AND CONCLUSIONS

Hypotheses

With regard to the digit naming task, all participants were expected to be slower to name digits in their second language than in their first language; this effect was expected to be especially prominent in unbalanced bilinguals. According to the results obtained, this prediction was verified. All E-S bilinguals were slower to name digits in Spanish, and this effect was significantly more prominent for the unbalanced E-S bilinguals. Although there was a slightly higher reaction time for S-E bilinguals naming digits in Spanish, the sample contained only balanced S-E bilinguals, almost all of whom had been taught math in English. To summarize, the results for the digit naming task showed that second language digit naming was slower than first language digit naming, and this effect was most prominent for the unbalanced bilinguals.

Examining the addition production task, problem size effects were found in L1 for both E-S and S-E bilinguals. When balanced E-S bilinguals produced answers to addition problems in English, they were faster when the problems were small than when the problems were large (mean RTs = 1136 and 1604 ms, respectively), which illustrated a traditional problem size effect. Although not significantly different, the balanced E-S bilinguals did exhibit somewhat larger problem size effects when answering in Spanish compared to answering in English (mean RTs = 1310 and 1810 ms, respectively), which was a prediction of the study. When looking at results for the unbalanced E-S bilinguals, there were significant differences, showing a greater problem size effect when answering in Spanish than when answering in English, which was also an expected outcome of the study. All S-E bilinguals in the study were balanced and also did not exhibit significant differences in problem size effects whether answering in L1 or L2, similar to the balanced E-S bilinguals.

The multiplication production task was also predicted to show larger problem size effects for L2 compared to L1. For the unbalanced E-S bilinguals, traditional problem size effects were observed along with significantly longer reaction times, especially for large problems, when producing multiplication answers in Spanish than when producing them in English. Balanced E-S bilinguals also showed larger problem size effects when answering in Spanish; however, the gap was not as large as for the unbalanced E-S bilinguals, and the S-E bilinguals showed traditional problem size effects; however no differences in reaction time were observed when comparing answers in English to answers in Spanish. Another prediction that was nicely illustrated in the data was that balanced E-S bilinguals and unbalanced E-S bilinguals showed no reaction time differences when producing multiplication answers in English, but there was a clear separation between them, with the unbalanced E-S bilinguals exhibiting significantly longer reaction times when answering in Spanish than the balanced E-S bilinguals.

The confusion verification task yielded interesting results. For true probes, everyone exhibited similar error rates when verifying true addition problems regardless of type of bilingual, categorization, or presentation format; however, when verifying true multiplication problems, significantly more errors were made for SNW presentation compared to ENW or digit presentation. Once again, this effect was found regardless of type of bilingual or categorization. Even S-E balanced bilinguals made significantly more errors verifying true multiplication problems when they were presented in SNW, which suggested that, even with extensive experience with and knowledge of Spanish number words (as illustrated in the three production tasks), S-E bilinguals had trouble retrieving multiplication answers from the retrieval network when presented with problems in SNW form.

False confusion probes were one of two types (neutral or confusion); Neutral probes contained answers that were ±1-3 away from the true answer, and confusion probes contained answers that were associatively related to the problem. Results for neutral probes indicated similar error rates for verifying false addition problems, regardless of presentation format; the same was true for verifying false multiplication problems, with the exception of SNW presentation, which exhibited significantly higher error rates than either digit or ENW presentations (refer to Figure 18a of Appendix 4). An example of a neutral multiplication probe would be siete x ocho = cincuenta y cinco (7 x 8 = 55). Confusion probes yielded the exact opposite result. Similar error rates were found for verifying false multiplication problems, regardless of presentation format; however, verifying false addition problems produced significantly higher error rates for SNW presentation than either digit or ENW presentation (refer to Figure 18b of Appendix 4). An example of a confusion addition probe would be siete + ocho = cincuenta y seis (7 + 8 = 56). As the above examples illustrate, both neutral multiplication probes and confusion addition probes were mainly paired with large Spanish number word answers. Even though balanced bilinguals had no trouble verbally producing Spanish number words for the digit naming and arithmetic tasks, they had difficulty when trying to verify answers to the same arithmetic problems presented as Spanish number words. This result speaks clearly to the idea of a preferred language for mathematics, which has been discussed in previous research, but was illustrated very clearly in this study, which also took careful consideration of type of bilingual and bilingual categorization. Over 90% of the current sample learned mathematics in English; therefore, even the fluent E-S and S-E bilinguals performed faster and more accurately when presented with arithmetic problems in familiar formats (e.g. digits and English number words) compared to Spanish number words, a format that they most likely had little to no experience with when performing arithmetic calculations. These results suggest that fluency in a language does not necessarily mean that the language would be integrated into the retrieval network used for numerical processing.

Additional Findings

One effect mentioned earlier regarding bilingual research was the preferred language effect. The preferred language effect occurs when bilinguals have a preferred language that they prefer to communicate in or, in the case of this study, use for mathematics. It has been found in previous research that bilinguals prefer to do math in the same language in which they learned math. Almost all of the participants in the current study learned math in English; however, the results for the digit naming, addition, and multiplication production tasks did not indicate a preferred language effect when the stimuli were presented as digits. For balanced bilinguals, no significant differences in reaction time were found that would indicate a preference for producing answers to arithmetic problems in English compared to Spanish. However, results from the confusion verification task did show significant reaction time differences for balanced bilinguals between verifying answers when the problems were presented as English number words than when they were presented as Spanish number words. This was the case for both balanced E-S and S-E bilinguals. Learning math in English resulted in a preference for the English number words compared to the Spanish number words, but did not have significant effects for digit presentation.

Another interesting finding from the study was found with regard to the problem size effect. The problem size effect predicts slower reaction times and higher error rates as the problem operands become larger. Traditional problems size effects were found for all three groups of bilinguals in the study; however, there was an interesting difference between balanced and unbalanced bilinguals with regard to this effect. For balanced bilinguals, problem size effects were found when they were producing answers in both English and Spanish; however, there were no significant reaction time differences between the two languages. In contrast, unbalanced bilinguals also showed evidence of problem size effects when answering in both English and Spanish, but for that group, there were significant reaction time differences between answering in English compared to answering in Spanish, with Spanish taking significantly longer. As fluency in the second language increased, retrieving and producing arithmetic answers in either language became easier, suggesting that numerical processing and the retrieval network were modified to account for second language acquisition.

Ties to the Literature

According to the Revised Hierarchical Model of bilingual memory (Kroll & de Groot, 1990), connections between L2 and the conceptual store should strengthen with increased fluency. In the case of this experiment, mathematical knowledge and retrieval would be included in the “conceptual store” of the RHM, and the results were in agreement with the ideas behind the model. For all production tasks (digit naming, addition, and multiplication), unbalanced bilinguals were slower to name digits as well as retrieve arithmetic answers in L2 (Spanish) compared to balanced bilinguals. Slower RTs in L2 indicate weaker connections between L2 (Spanish) and mathematical concepts requiring translation before answers could be produced. In contrast, the results of balanced bilinguals showed similar reaction times whether producing answers in L1(English) or L2(Spanish), indicating that, for balanced bilinguals, connections to mathematical knowledge were strong in both languages. To summarize, the preditions of the RHM were supported in the results from this experiment and generalize the conceptual store to include mathematical cognition and math knowledge.

According to the Bilingual Interactive Activation (BIA) model and its succesor, the BIA+ model (Dijkstra & Van Heuven, 1998; 2002), both languages are activated in parallel as a word string enters into the system. The BIA+ model theorized that, in addition to orthographic representations, phonemic and semantic representations are also activated in both languages as the word string moves up the system until the individual can positively identify the presented word is an L1 or an L2 word, and that this process should be relatively fast. The results of the current study indicated that the predictions for the BIA and BIA+ models were consistent with regard to numerical processing and math knowledge, but only under two conditions. Firstly, bilinguals needed to be fluent in both languages (balanced); unbalanced bilingual performance in the second language (Spanish) indicated longer reaction times for both production and verification tasks (refer to Figures 3, 4b, 6, 8, and 12 in Appendix 4). This indicated that, for unbalanced bilinguals performing arithmetic, the two languages were not activated in parallel, as the BIA and BIA+ models would have suggested. Secondly, even for balanced S-E bilinguals, the arithmetic stimuli needed to be presented in digit form as opposed to either English or Spanish number words. For example, the addition and the multiplication production tasks showed that, for digit presentation, problem size effects were found in both L1(English) and L2(Spanish) for balanced and unbalanced bilinguals; however, when examining reaction times, those effects were not significantly different for balanced bilinguals (i.e. although problem size effects existed in both languages, no significant differences in reaction time were found between producing the answer in L1 or L2). The BIA and BIA+ models did not make predictions about bilinguals solving arithmetic problems, but if the idea behind the models was correct, then both languages would be activated during feature detection even though a number string was presented to the system. Application of the models to arithmetic processing may look something like the following: for balanced bilinguals, the number string enters the system as digits although it is likely that both languages are also activated in parallel as the features of the digits are being detected. Once the number string was processed and the answer retrieved from the retrieval network, both languages would still be available for balanced bilinguals to produce the answer (explaining the lack of significant reaction time differences between L1 and L2). In that case, the problem size effects were occurring under the guise of the original theory of a retrieval network (Ashcraft & Battaglia, 1978). In contrast, unbalanced bilinguals did show significant reaction time differences in problem size effects when producing answers in L1 compared to L2. Here, the number string entered into the system with only the dominant L1 activated. Once the answer was determined, unbalanced bilinguals then had to translate the answer into L2 to be able to produce the answer. That particular result indicated that the BIA and BIA+ models were designed with the idea that bilinguals were fluent in both languages and did not take into account what happened as individuals were in the process of learning an L2 or if they were unbalanced in L2. The BIA and BIA+ models also could not account for the confusion findings in which fluent bilinguals exhibited higher error rates when solving arithmetic problems presented in Spanish number words (refer to Figures 18a & 18b in Appendix 4) compared to digits or English number words. Even though S-E bilinguals were fluent in both languages and learned Spanish as their first language, it was the language that they learned math in that determined performance on simple arithmetic tasks. In sum, under certain conditions, the BIA and BIA+ models generalized to the numerical cognition and mathematical knowledge of bilinguals. For example, with regard to numerical processing, at least when balanced bilinguals were presented with arithmetic stimuli in digit form, results indicated that both languages did activate in parallel as the stimuli moved through the system as the models would suggest. However, without the above caveats, the BIA and BIA+ models were to general to be directly applied to the numerical cognition of bilinguals.

Another important aspect of this project was to examine whether or not the ideas behind the encoding-complex model (Campbell & Epp, 2004) proposed for Chinese-English bilinguals would generalize to a Spanish-English population. The model could be considered a special case of the RHM (Kroll & de Groot, 1997) in that stronger and weaker connections are proposed between systems; however, the encoding-complex model was designed specifically in reference to number processing. One difference between the Campbell and Epp (2004) sample and the sample for this study was that Spanish did not have a different number format like Mandarin; however, after looking at the general assumptions of that model, it could be adapted to Spanish-English bilinguals as well. Two key ideas from the encoding-complex model were that task-specific activation of information would activate one or more representational codes. For example, it would predict that when performing arithmetic operations or naming digits, activation of both number words as well as digits would occur. Also, the model predicted interactive processes (e.g. interactive communication between the Arabic and the number word components of the model) that could be influenced with task-specific practice to optimize resistance to interference from irrelevant information. In terms of mathematics and arithmetic, encoding-retrieval processes would become more efficient when stimuli were presented in a familiar format that had been well practiced. Regarding the current study, results from the addition and multiplication tasks indicated interactive processes between languages (at least in the case of balanced bilinguals); this was illustrated in the form of similar reaction times when answering in English or Spanish, possibly indicating interactive activation for the production of both English and Spanish number words, given familiar (Arabic digit) stimuli. This result made sense considering the sample of balanced bilinguals in the study reported speaking both languages often. The confusion task also yielded results that were in accord with the model assumptions. For both true and false probes, as well for both neutral and confusion probes, longer reaction times and higher error rates were observed for verifications to addition and multiplication problems that were presented in Spanish number words. Considering almost all of the participants in the study learned math in English, they would be more familiar with Arabic as well as

English number word presentations because they would have received more task-specific practice in those formats. Although balanced bilinguals were used to speaking both languages often (hence, they were able to produce number words in either language with the same speed and accuracy), their arithmetic facts were more easily retrieved when presented as either digits or English number words, which was most likely a result of task-specific practice as suggested by the encoding complex model. A modified version of the encoding-complex model is given in Figure 9.

General Conclusions

In the past, bilingual research has typically investigated samples of either all balanced (Campbell & Epp, 2004; Duyck & Brysbaert, 2002; Ellis, 1992; Marsh & Maki, 1976; Vaid & Frenck-Mestre, 1991) or all unbalanced (Frenck-Mestre & Vaid, 1993; Spelke & Tsivkin, 2001) bilinguals, with the exception of Duyck and Brysbaert (2004), which looked at a bilingual sample containing both. Also, the criterion for determining categorization (balanced vs. unbalanced) was never standardized, and each study had its own way of determining categorization. The bulk of bilingual research has been done using verbal stimuli to help develop models of the bilingual language system, and few studies have examined the bilingual language system in terms of math cognition and numerical processing. The purpose of this dissertation study was to design a bilingual study that took careful account of bilingual categorization while examining performance on several well-known mathematical tasks. Using a standardized language questionnaire (the LEAP-Q), the study was able to more reliably categorize the bilinguals in the study as either balanced or unbalanced in their second language. The results from this study would predict that, without arithmetic practice, bilinguals will have a hard time performing arithmetic operations in the language not used for math even though producing number words in that language (when presented with a familiar form) is not affected. The results also tied in well with previous models of bilingual memory, such as the RHM, as well as the encoding-complex model, a more current model of bilingual numerical processing. Even though Spanish did not have a separate format for numerals, and almost all of the sample learned math in English it was possible to adapt the encoding-complex model to balanced E-S and S-E bilinguals only adjusting for some of the connections between systems to be weaker or stronger than those presented with the original model of Chinese-English. It may be possible to adapt the encoding-complex model to most bilingual numerical processing as long as numerical formats and the language that the sample learned math in are taken into account.

Future research could take another look at balanced S-E bilinguals who learned all of their arithmetic in Spanish to see if they perform differently given Spanish number word presentation. It would be especially interesting to compare them to balanced S-E bilinguals who learned all of their arithmetic in English. The results from the current study would predict similar performance on production tasks as well as the confusion task; however, those bilinguals who learned arithmetic in Spanish would be predicted to have higher error rates for English number word presentation, and those who learned arithmetic in English would be predicted to have higher error rates for Spanish number words. It may also be interesting to design an experiment in which bilinguals are given task-specific practice with arithmetic presented in number words from the language in which they did not learn math to see if some of the effects from the confusion task here would be diminished. The encoding-complex model may predict so, since there is room for the connections between systems to become stronger.

APPENDIX 1

OPRS-APPROVAL

UNLV
UNIVERSITY OF NEVADA LAS VEGAS

Approved
JUL 06 2010
Expires
JUL 05 2011

Social/Behavioral IRB – Expedited Review
Approval Notice

NOTICE TO ALL RESEARCHERS:

Please be aware that a protocol violation (e.g., failure to submit a modification for any change) of an IRB approved protocol may result in mandatory remedial education, additional audits, re-consenting subjects, researcher probation suspension of any research protocol at issue, suspension of additional existing research protocols, invalidation of all research conducted under the research protocol at issue, and further appropriate consequences as determined by the IRB and the Institutional Officer.

DATE: July 9, 2010

TO: **Dr. Mark Ashcraft**, Psychology

FROM: Office of Research Integrity - Human Subjects

RE: Notification of IRB Action by Dr. Ramona Denby Brinson, Chair
Protocol Title: **Bilingualism and Math Cognition**
Protocol #: 1005-3477

This memorandum is notification that the project referenced above has been reviewed by the UNLV Social/Behavioral Institutional Review Board (IRB) as indicated in Federal regulatory statutes 45 CFR 46. The protocol has been reviewed and approved.

The protocol is approved for a period of one year from the date of IRB approval. The expiration date of this protocol is July 5, 2011. Work on the project may begin as soon as you receive written notification from the Office of Research Integrity - Human Subjects (ORI Human Subjects).

PLEASE NOTE:
Attached to this approval notice is the **official Informed Consent/Assent (IC/A) Form** for this study. The IC/A contains an official approval stamp. Only copies of this official IC/A form may be used when obtaining consent. Please keep the original for your records.

Should there be *any* change to the protocol, it will be necessary to submit a **Modification Form** through ORI Human Subjects. No changes may be made to the existing protocol until modifications have been approved by the IRB.

Should the use of human subjects described in this protocol continue beyond July 5, 2011, it would be necessary to submit a **Continuing Review Request Form** *60 days* before the expiration date.

If you have questions or require any assistance, please contact the Office of Research Integrity - Human Subjects at IRB@unlv.edu or call 895-2794.

Office of Research Integrity – Human Subjects
4505 Maryland Parkway • Box 451047 • Las Vegas, Nevada 89154-1047

APPENDIX 2

BILINGUAL MEMORY MODELS

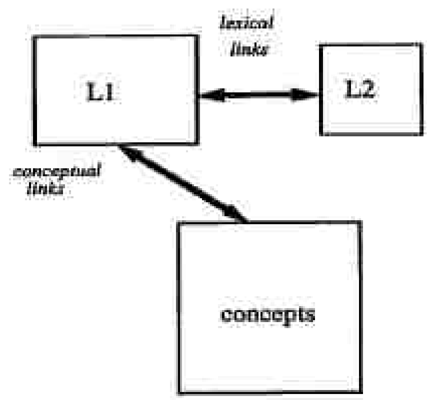

Figure 1. Word Association Model taken from Potter (1984)

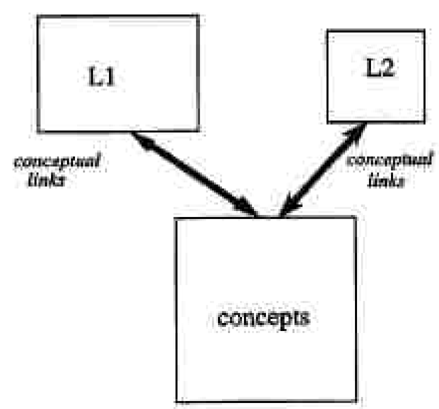

Figure 2. Concept Mediation Model taken from Potter (1984)

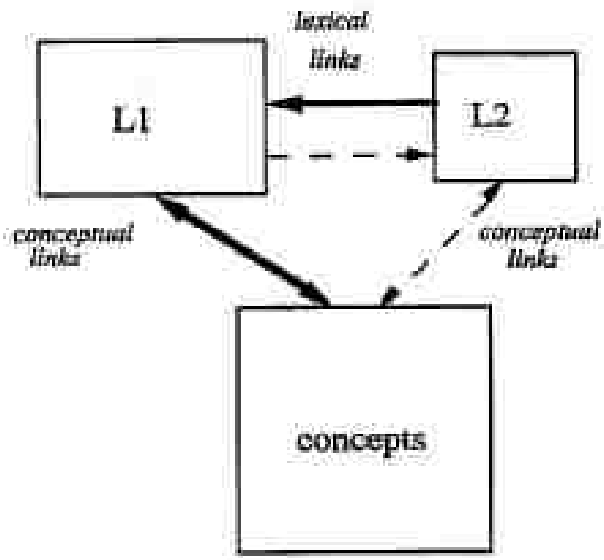

Figure 3. Revised Hierarchical Model of Bilingual Memory (RHM) taken from Kroll and de Groot (1997)

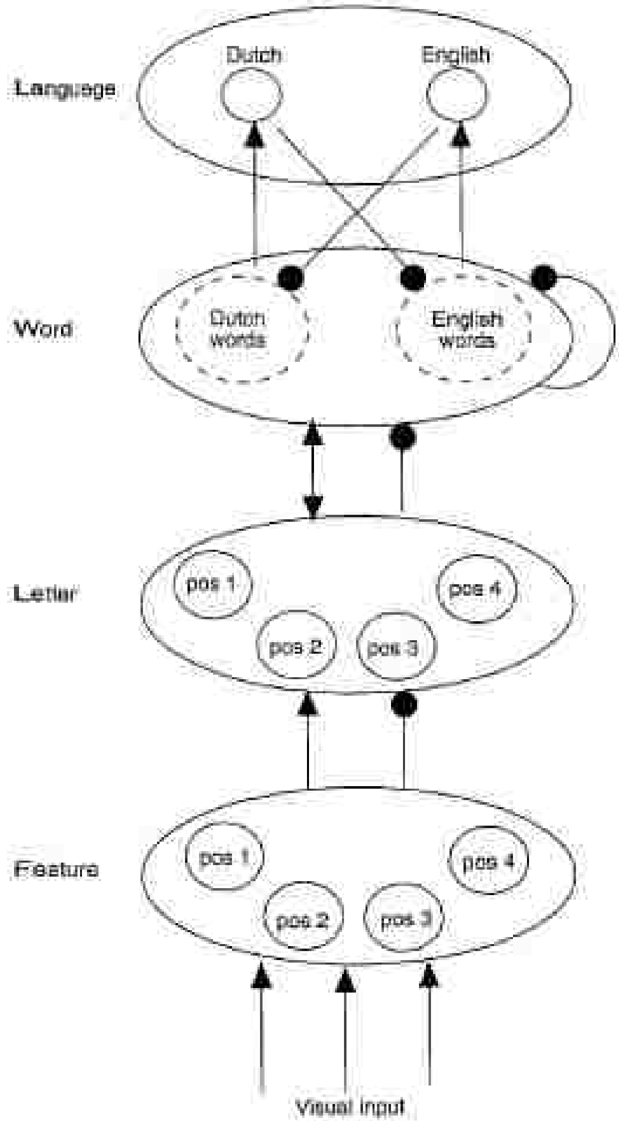

Figure 4. Bilingual Interactive Activation Model (BIA) taken from Dijkstra and Van Hueven (2002)

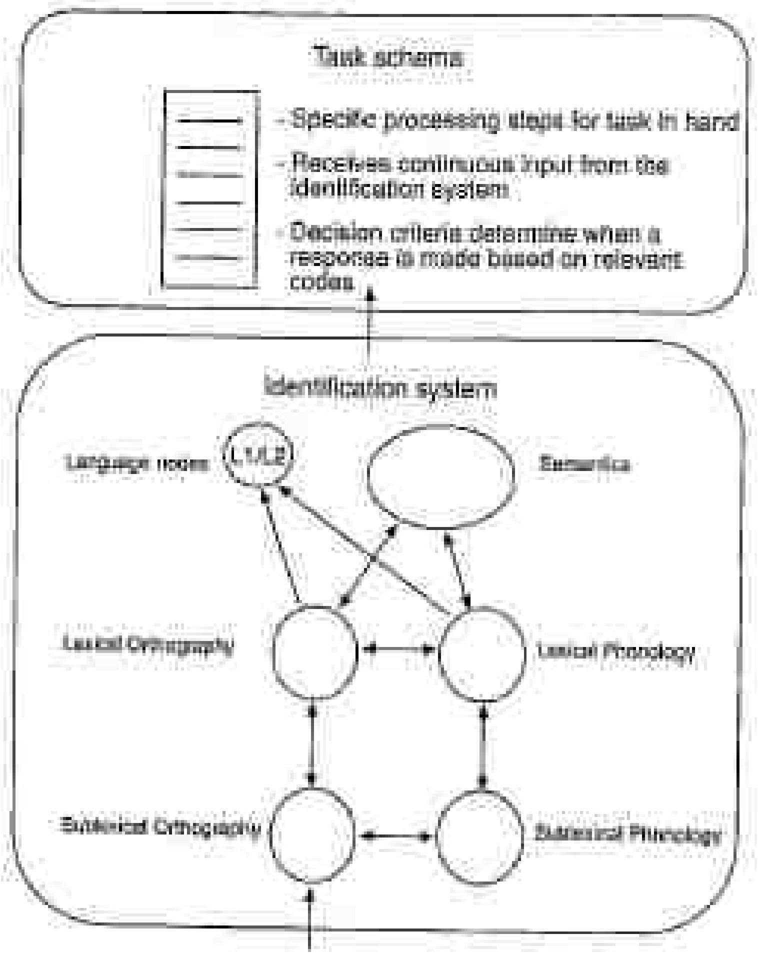

Figure 5. Bilingual Interactive Activation Plus (BIA+) model taken from Dijkstra and Van Hueven (2002)

APPENDIX 3

NUMERICAL PROCESSING MODELS

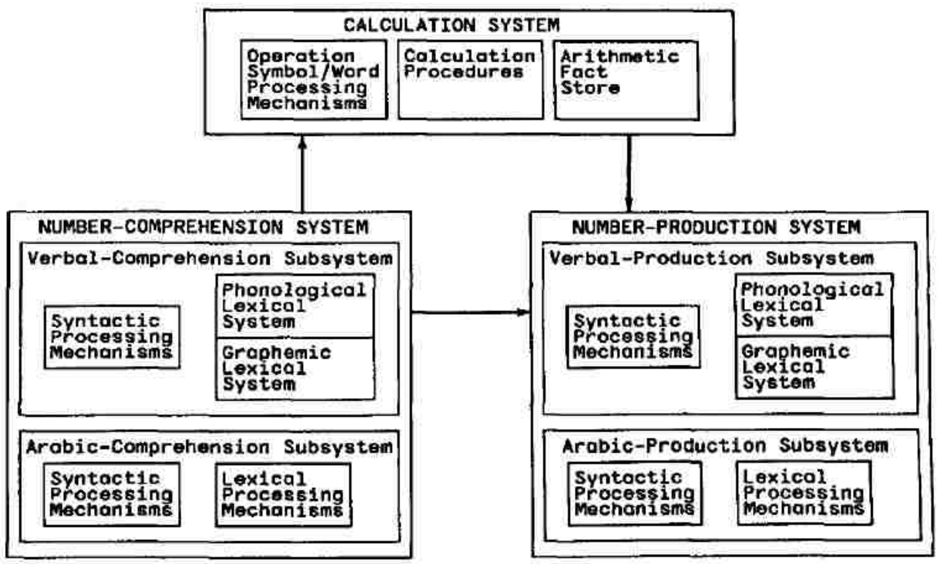

Figure 6. Number-processing model of McCloskey, Caramazza, and Basili (1985) adapted with permission and taken from Campbell and Clark (1988).

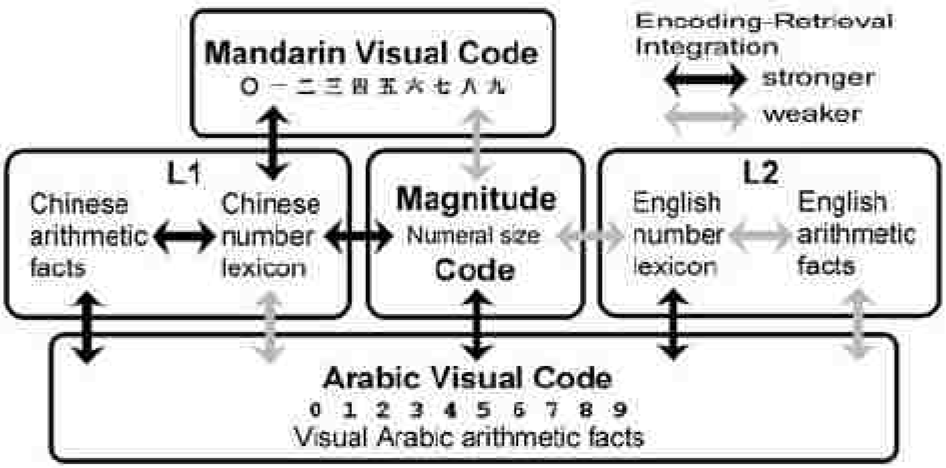

Figure 7. Encoding Complex Model for cognitive number processing extended to Chinese-English bilinguals taken from Campbell and Epp (2004)

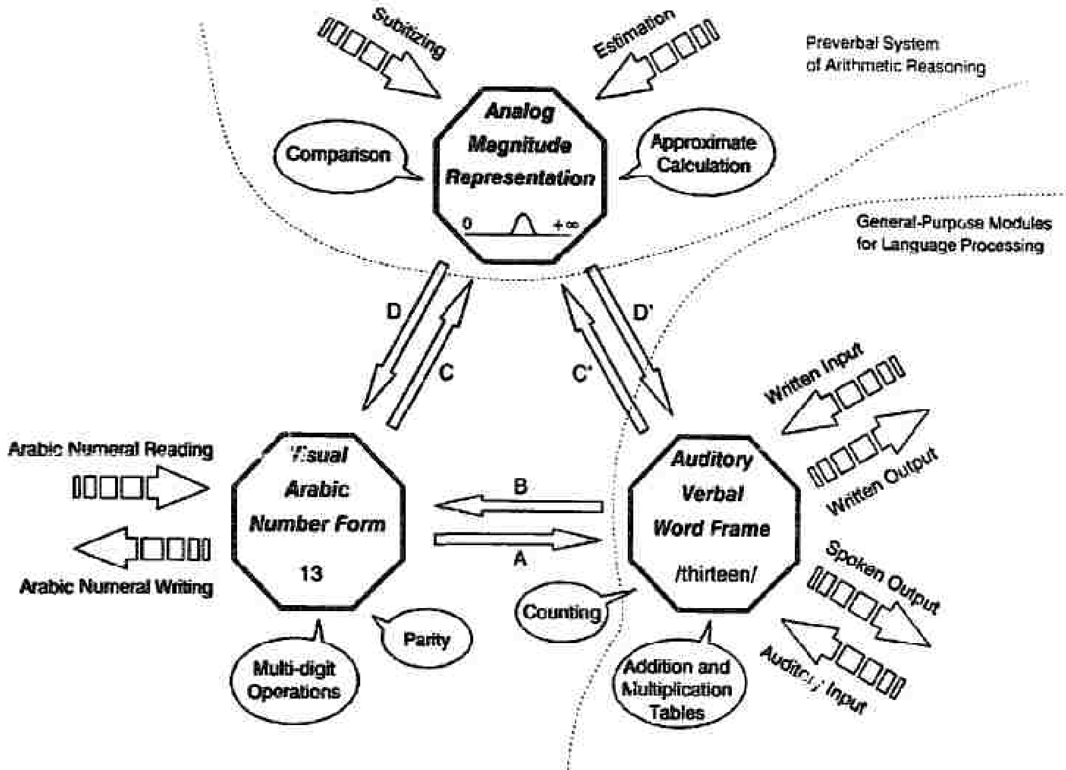

Figure 8. Triple Code Model of cognitive number processing taken from Dehaene (1992)

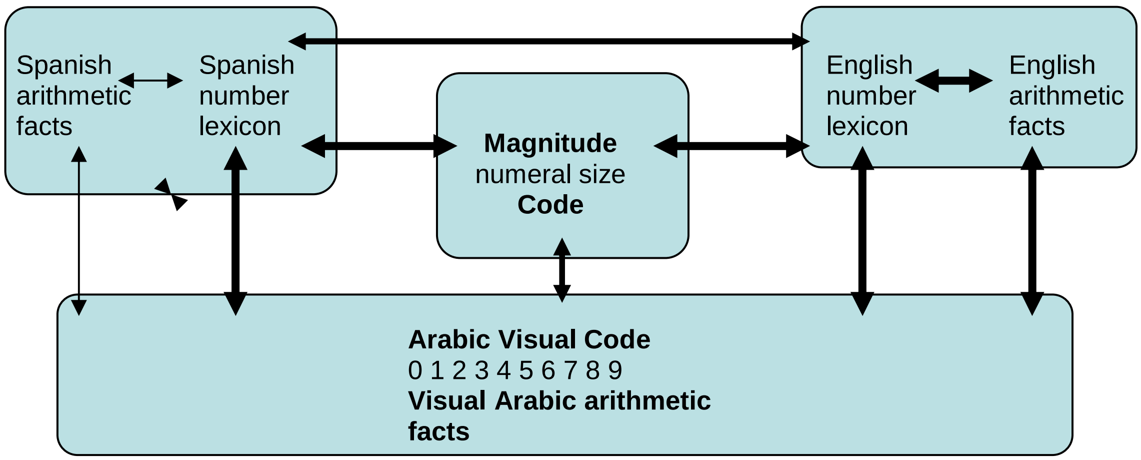

Figure 9. An encoding-complex model adapted to Spanish-English bilinguals. Bold lines indicate stronger connections between systems.

APPENDIX 4

TABLES AND FIGURES

Table 1

A breakdown of participants for each experimental task.

	Type of Bilingual					
	English-Spanish			Spanish-English		
Task	**Total**	**Balanced**	**Unbalanced**	**Total**	**Balanced**	**Unbalanced**
LEAP-Q	37	25	12	43	43	0
Demographics	37	25	12	41	41	0
Digit Naming	28	19	9	37	37	0
Addition Production	28	19	9	36	36	0
Multiplication Production	24	17	7	25	25	0
Confusion Verification	37	25	12	38	38	0

Table 2

The means and standard deviations of the findings from the demographic sheet and LEAP-Q

	Type of Bilingual	
Categorization		
	English-Spanish	**Spanish-English**
Balanced	25	43
Unbalanced	12	0
Demographic Variable		
Gender M/F	14/23	12/29
Age	20.11(3.733)	20.49(5.358)
Ranking	2.08(1.115)	1.83(1.181)
MathLang E/S	37/0	28/15
# High School Math Classes Taken	4.12(.545)	4.02(.880)
High School MathLang E/S	37/0	40/3
Ethnic Group: % of Total		
African American	2.7	N/A
Hispanic/Latino	40.5	97.6
Native American	N/A	N/A
Asian/Pacific Islander	10.8	2.4
Caucasian	32.4	N/A
Other	13.5	N/A

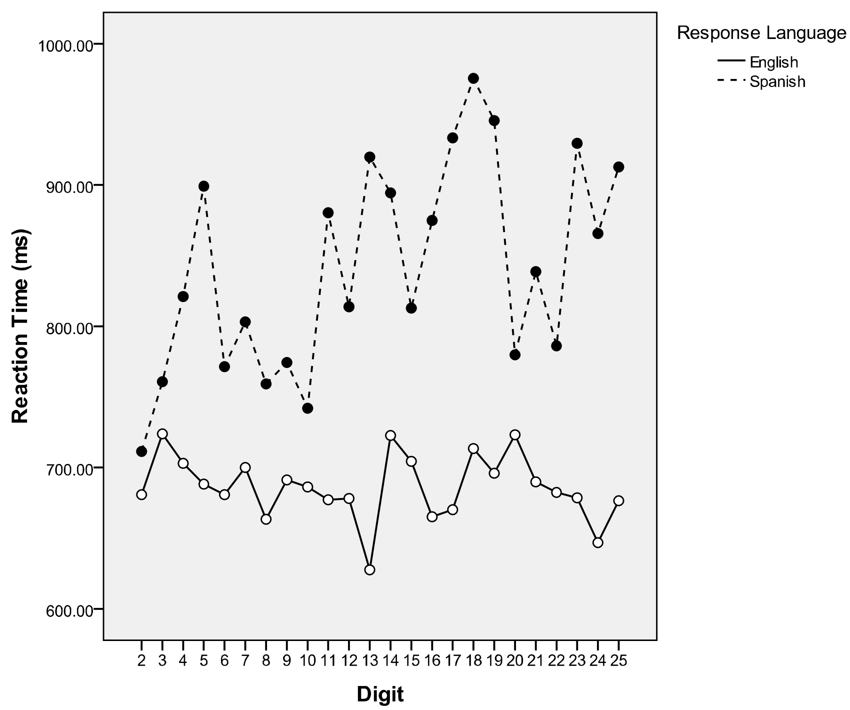

Figure 1. Results for the digit x response language interaction during the digit naming task using the full sample of participants

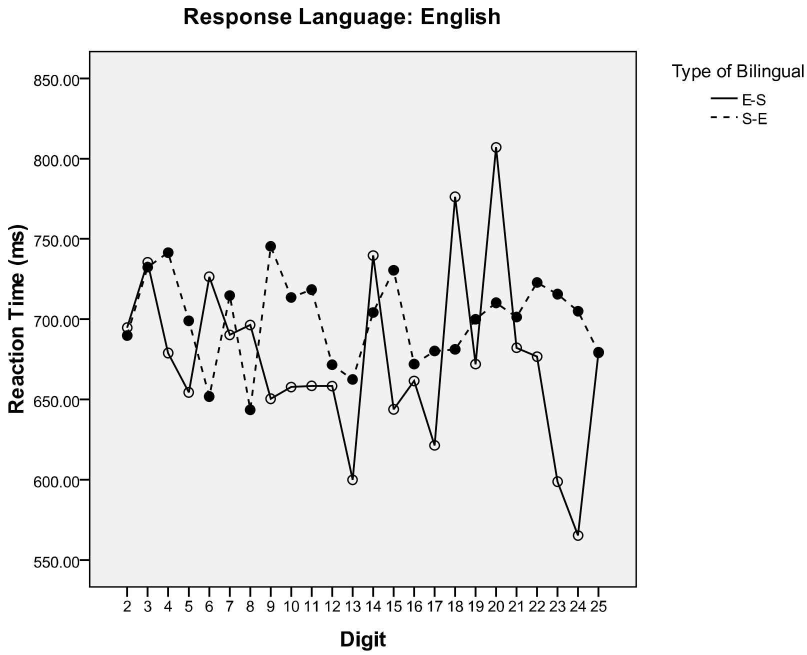

Figure 2a. Results for the response language x digit x type of bilingual interaction for the digit naming task using only balanced bilinguals

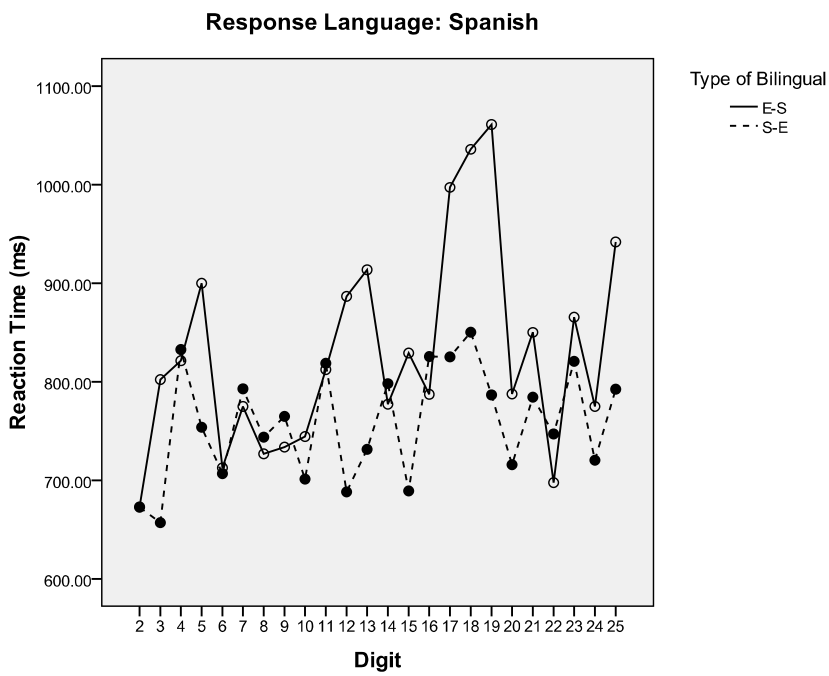

Figure 2b. Results for the response language x digit x type of bilingual interaction for the digit naming task using only balanced bilinguals

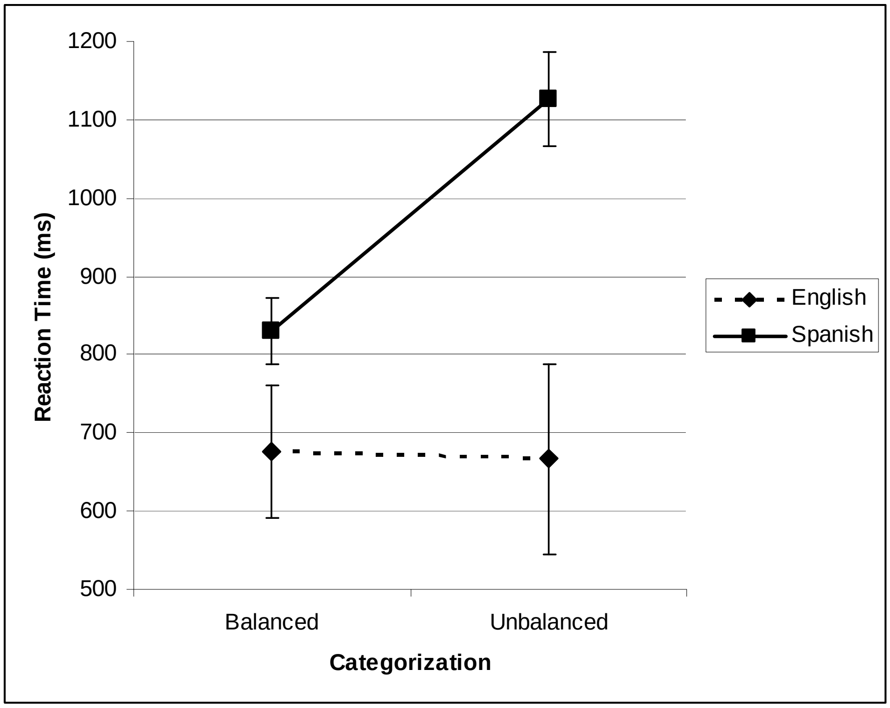

Figure 3. Results for the response language x categorization interaction for the digit naming task using only E-S bilinguals

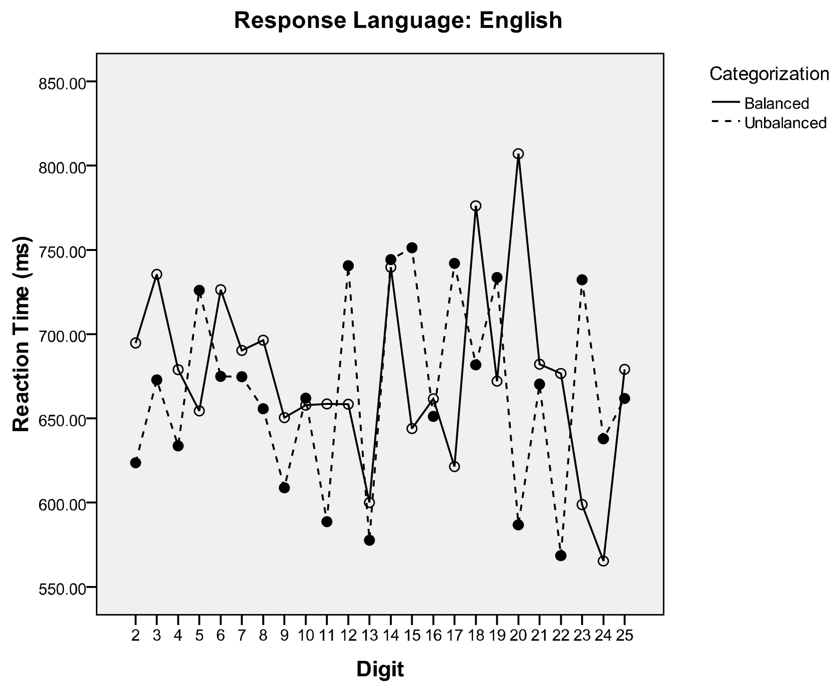

Figure 4a. Results for the response language x digit x categorization interaction for the digit naming task using only E-S bilinguals

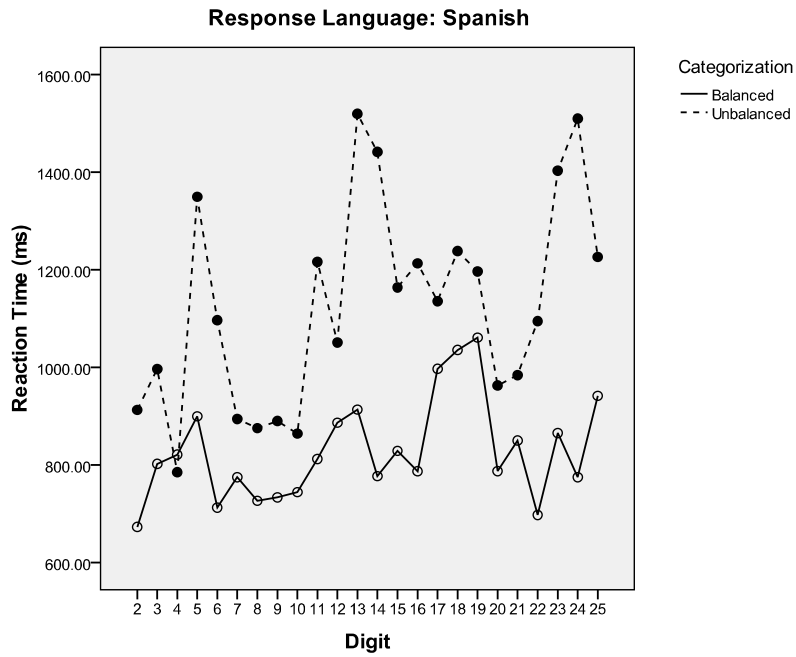

Figure 4b. Results for the response language x digit x categorization interaction for the digit naming task using only E-S bilinguals

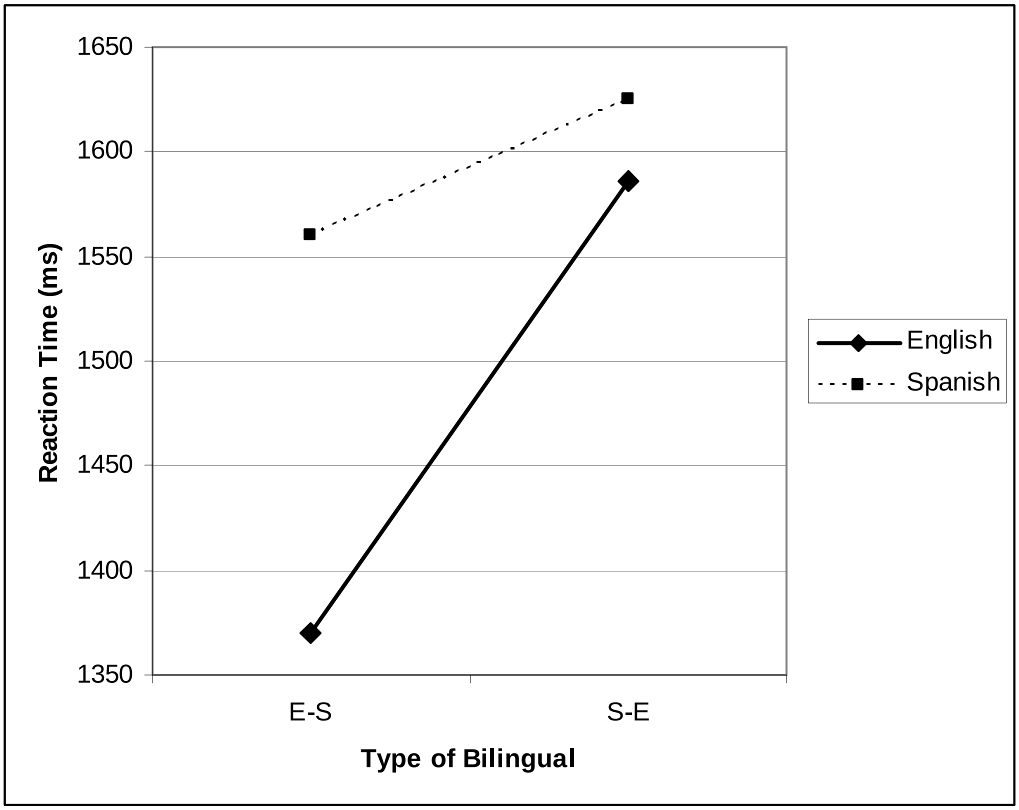

Figure 5. Results for the response language x type of bilingual interaction for the addition production task using only balanced bilinguals

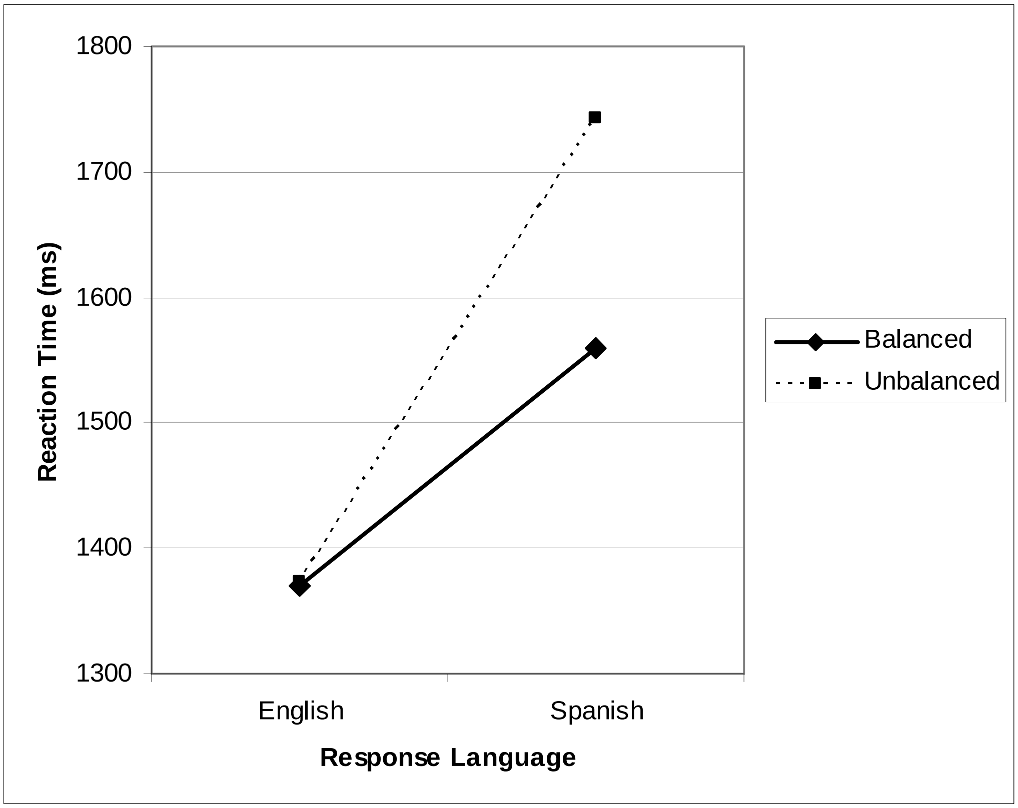

Figure 6. Results for the response language x categorization interaction for the addition production task using only E-S bilinguals

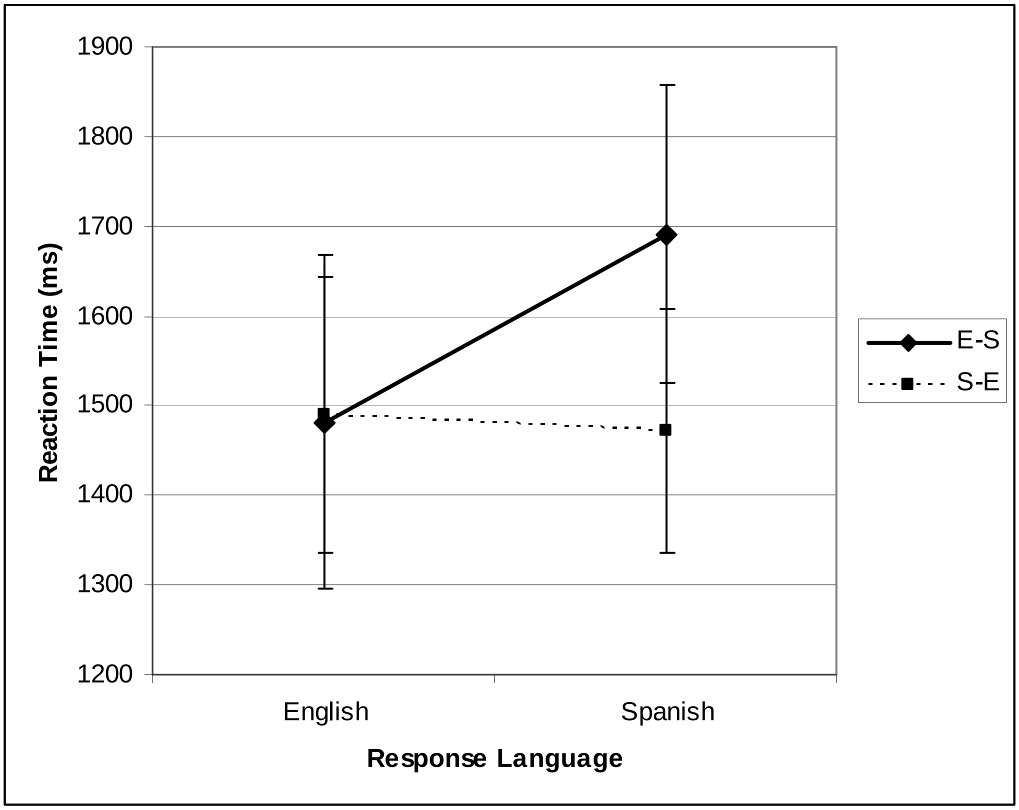

Figure 7. Results for the response language x type of bilingual interaction for the multiplication production task using only balanced bilinguals

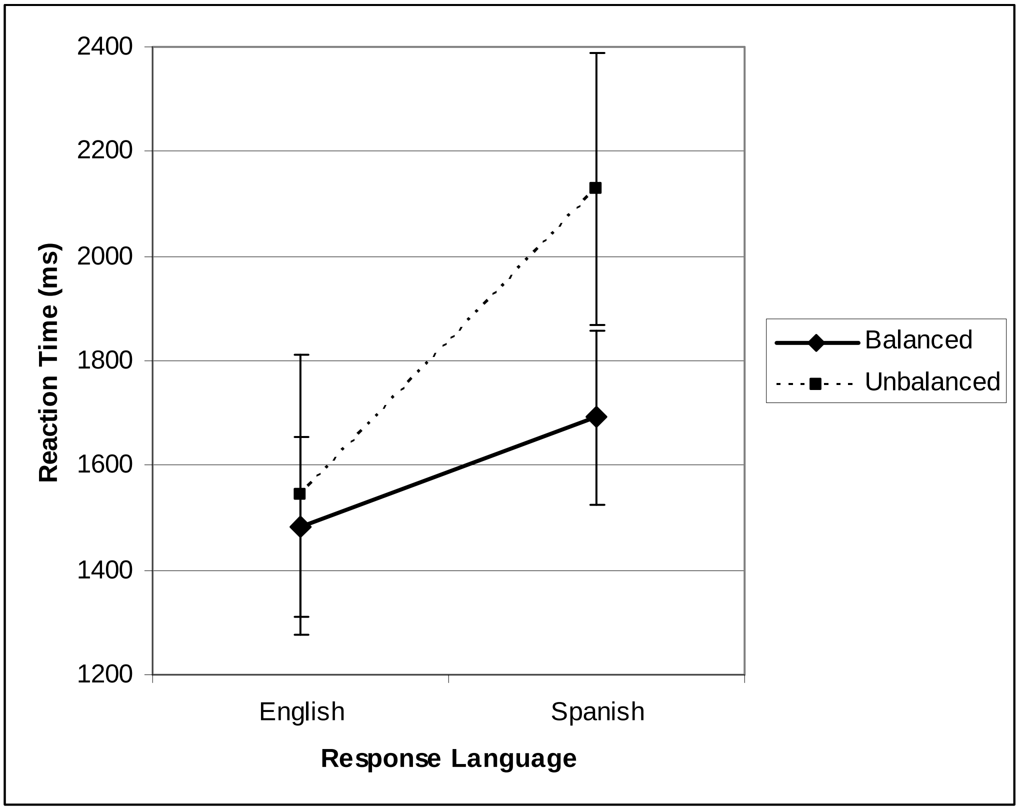

Figure 8. Results for the response language x categorization interaction for the multiplication production task using only E-S bilinguals

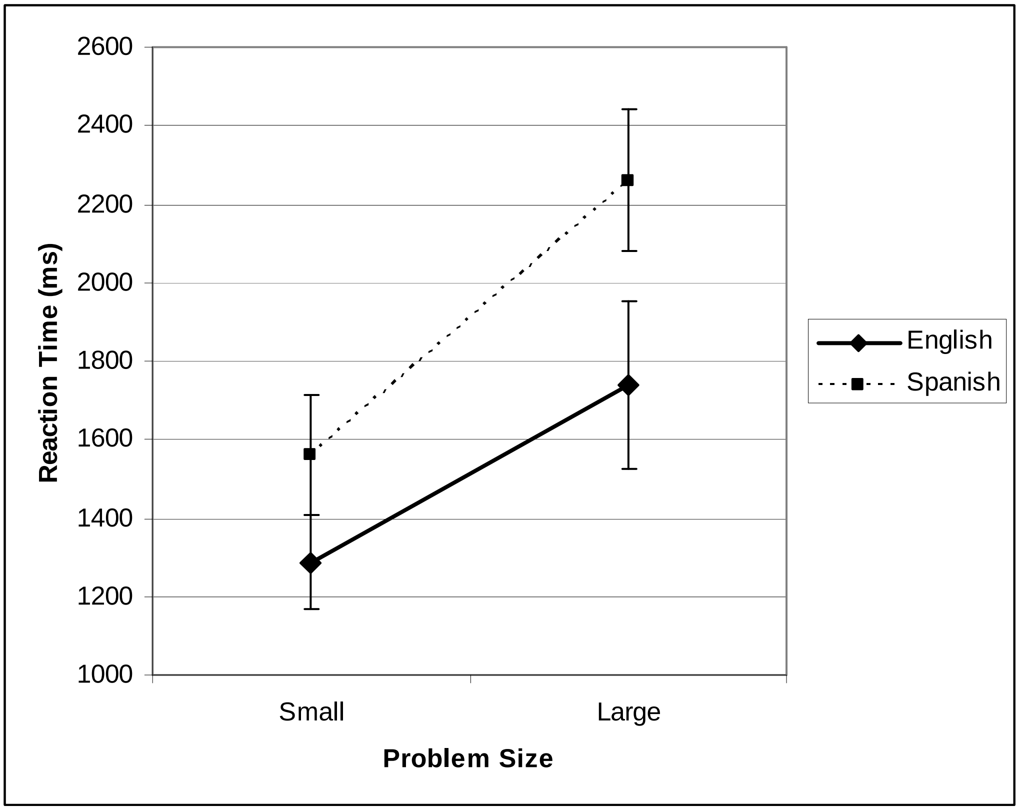

Figure 9. Results for the problem size x response language interaction for the multiplication production task using only E-S bilinguals

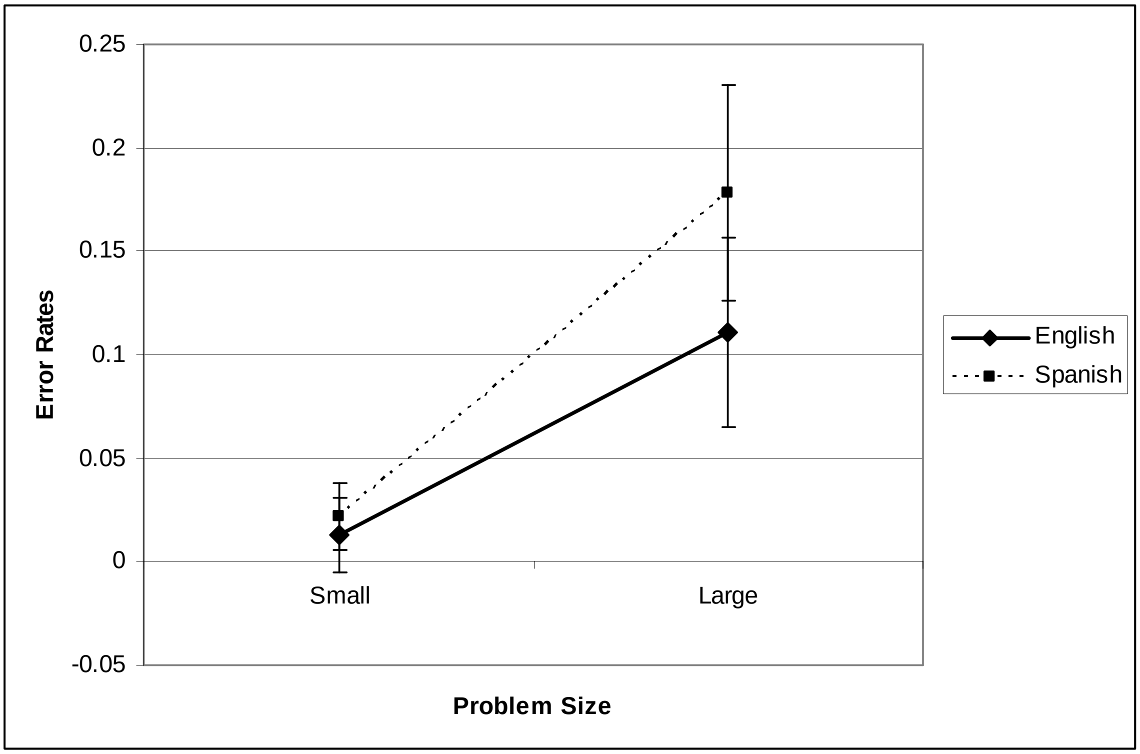

Figure 10. Results for the problem size x response language interaction for the multiplication production task using only E-S bilinguals

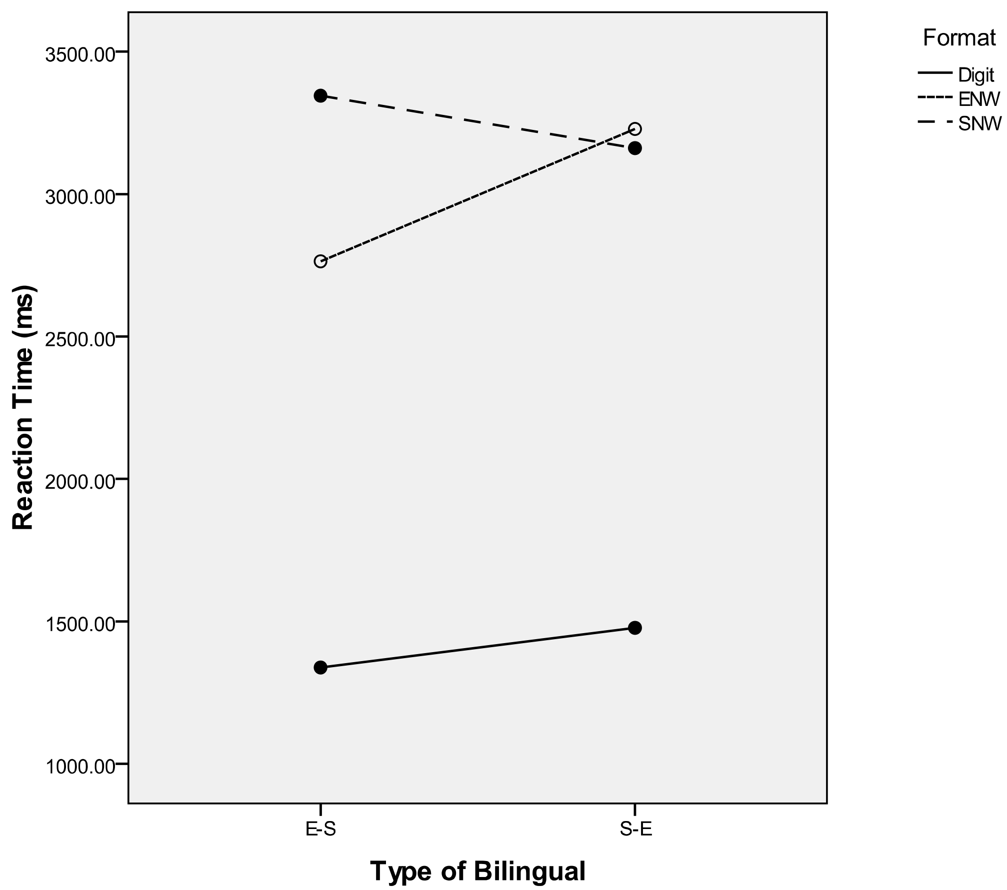

Figure 11. Results for the format x type of bilingual interaction for true probes from the confusion verification task using the full sample

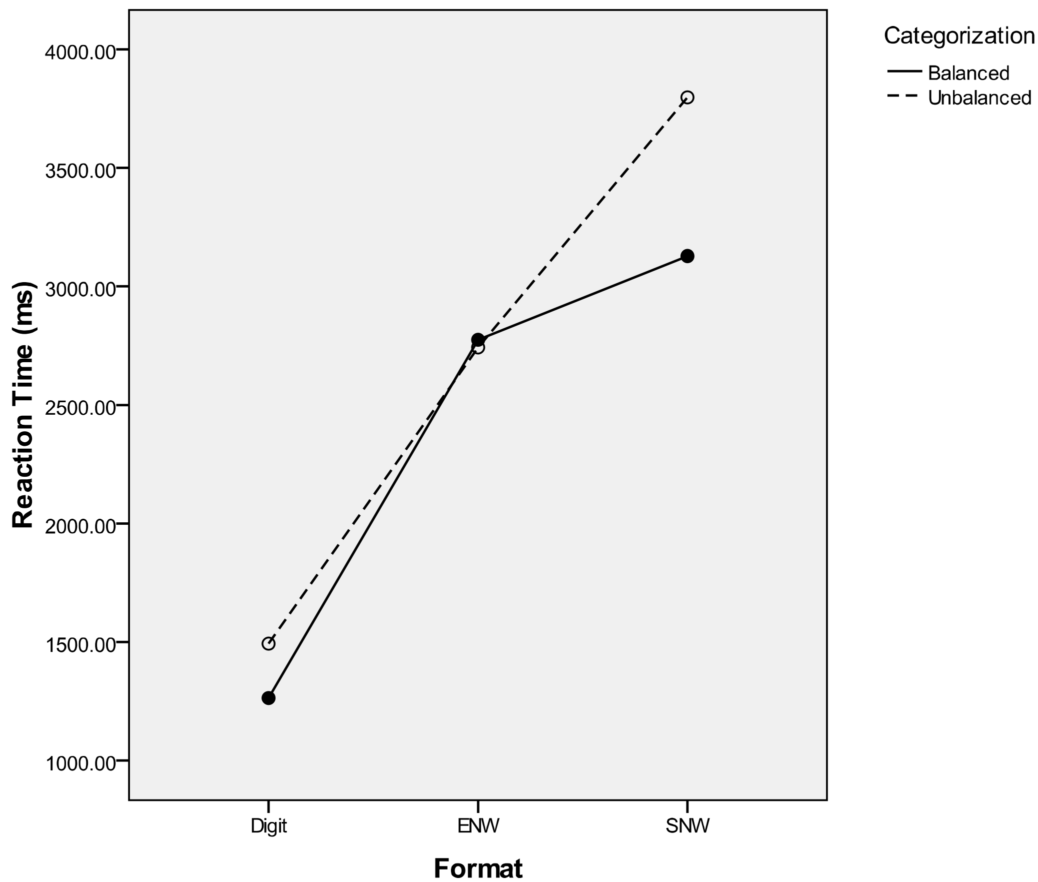

Figure 12. Results for the format x categorization interaction for true probes from the confusion verification task using only E-S bilinguals

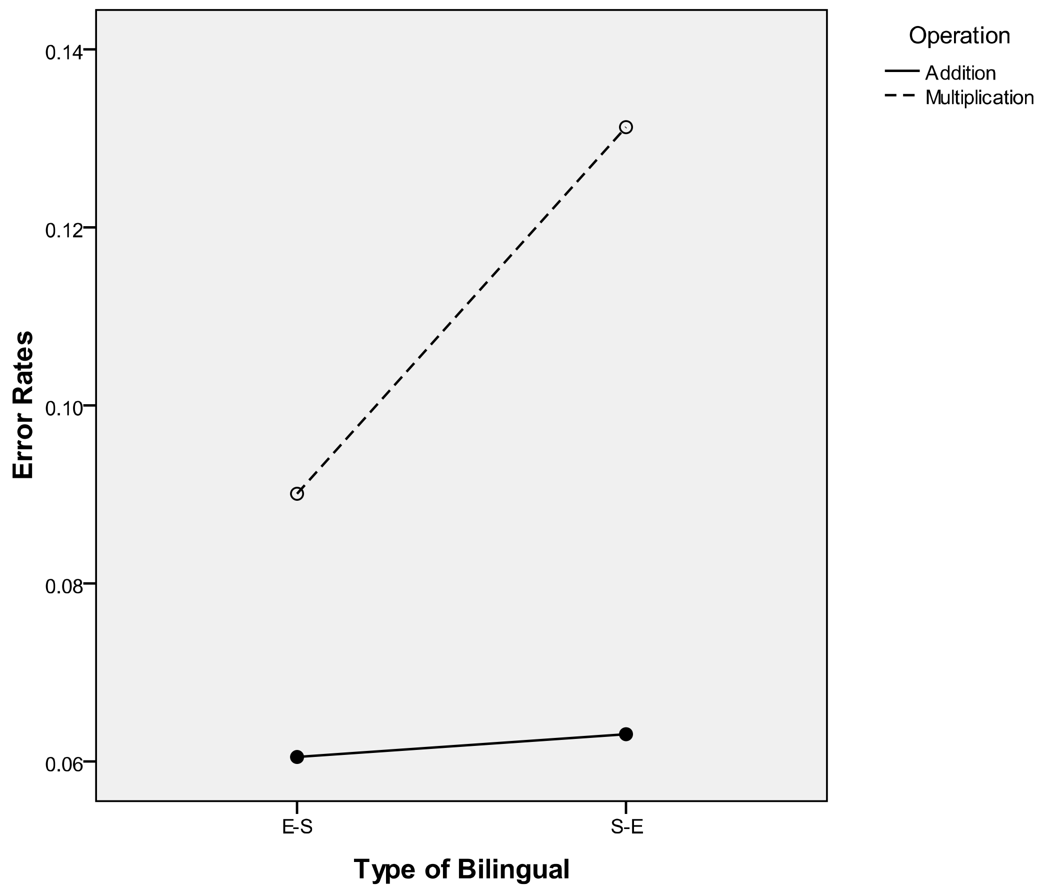

Figure 13. Results for the operation x type of bilingual interaction for true probes from the confusion verification task using the full, three-group, sample

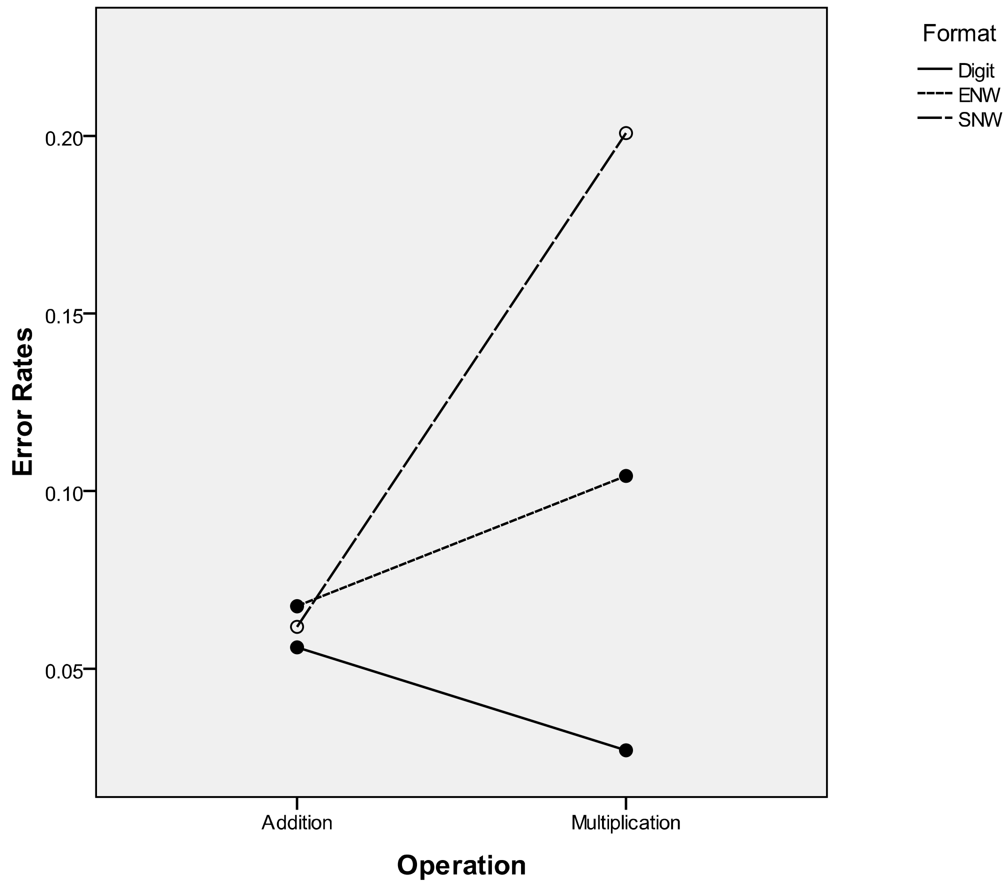

Figure 14. Results for the operation x format interaction for true probes from the confusion verification task using the full, three-group, sample

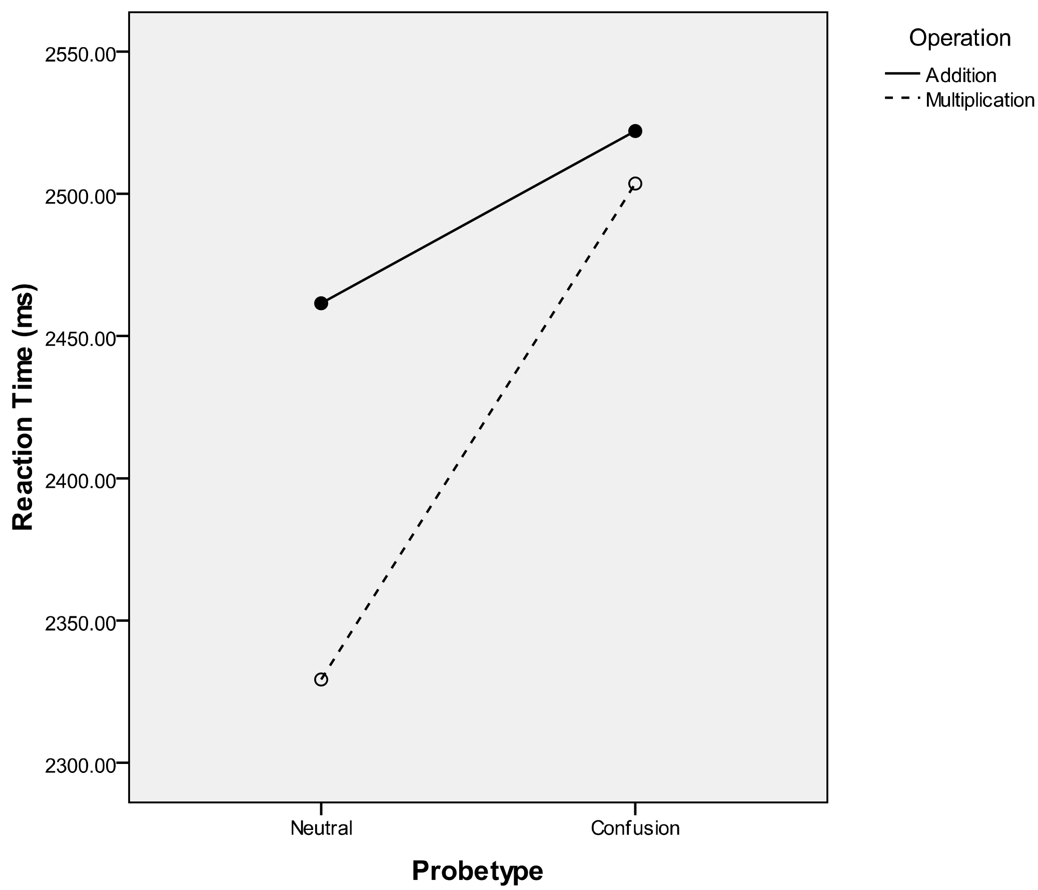

Figure 15. Results for the probe type x operation interaction for false probes from the confusion verification task using the full, three-group, sample

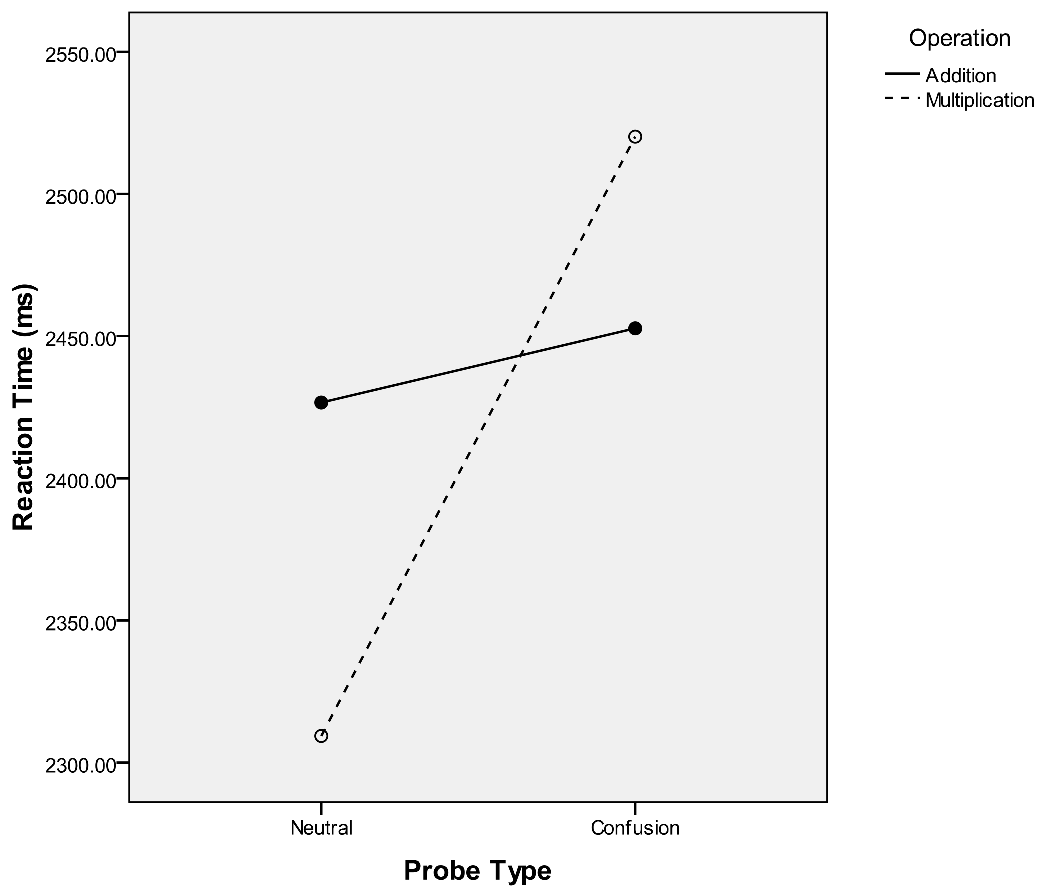

Figure 16. Results for the probe type x operation interaction for false probes from the confusion verification task using only E-S bilinguals

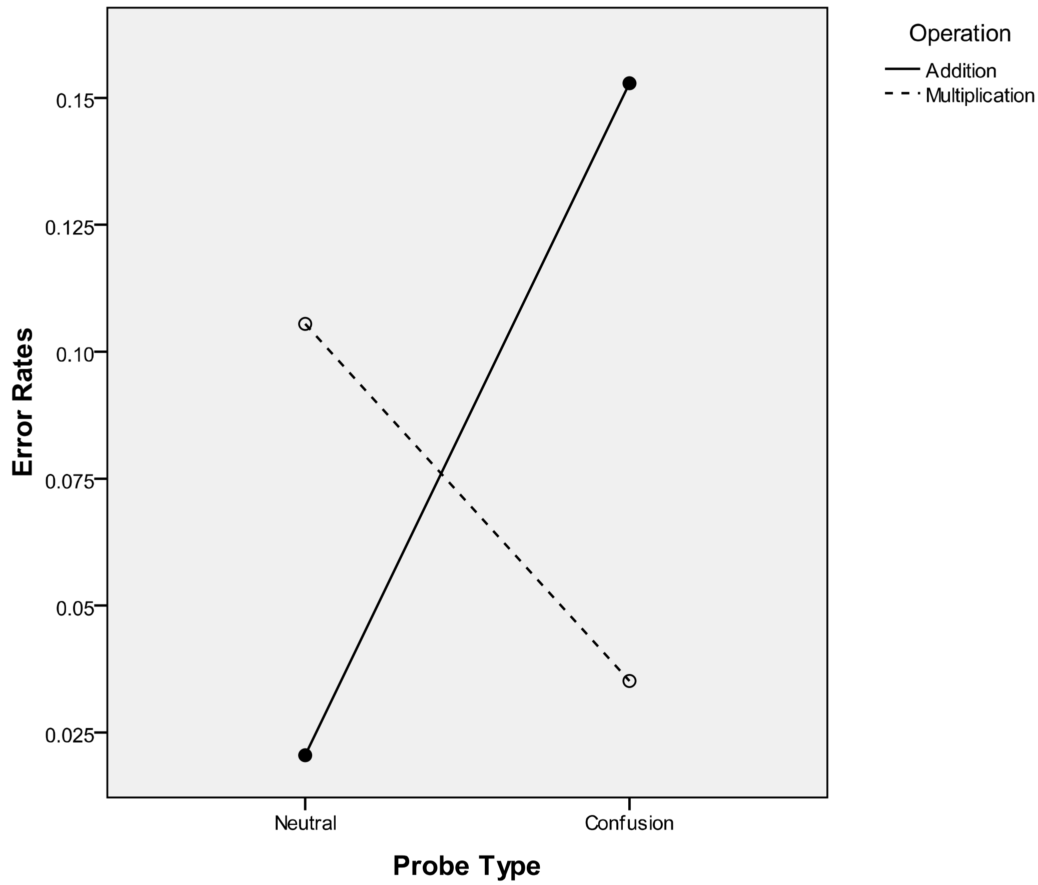

Figure 17. Results for the probe type x operation interaction for false probes from the confusion verification task using only balanced bilinguals

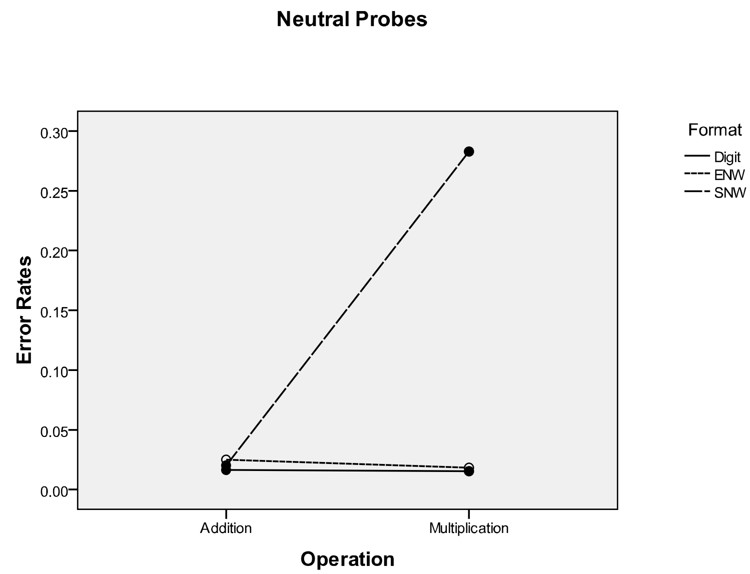

Figure 18a. Results for the format x operation interaction for neutral false probes from the confusion verification task using only balanced bilinguals

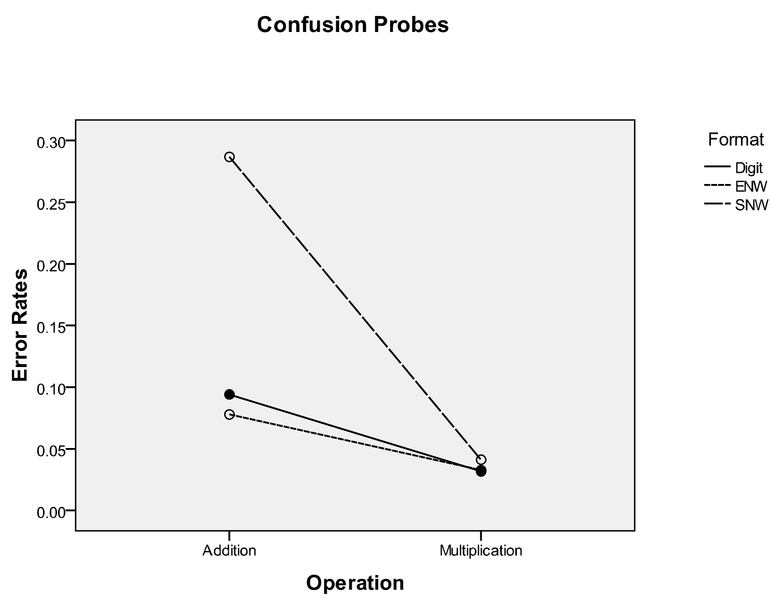

Figure 18b. Results for the format x operation interaction for confusion false probes from the confusion verification task using only balanced bilinguals

APPENDIX 5

LEAP-Q

Northwestern Bilingualism & Psycholinguistics Research Laboratory

Please cite Marian, Blumenfeld, & Kaushanskaya (2007). The Language Experience and Proficiency Questionnaire (LEAP-Q): Assessing language profiles in bilinguals and multilinguals. *Journal of Speech Language and Hearing Research, 50* (4), 940-967.

Language Experience and Proficiency Questionnaire (LEAP-Q)

Last Name		First Name		Today's Date	
Age		Date of Birth		Male ☐	Female ☐

(1) Please list all the languages you know **in order of dominance**:

1	2	3	4	5

(2) Please list all the languages you know **in order of acquisition** (your native language first):

1	2	3	4	5

(3) Please list what percentage of the time you are *currently* and *on average* exposed to each language.
(*Your percentages should add up to 100%*):

List language here:					
List percentage here:					

(4) When choosing to read a text available in all your languages, in what percentage of cases would you choose to read it in each of your languages? Assume that the original was written in another language, which is unknown to you.
(*Your percentages should add up to 100%*):

List language here					
List percentage here:					

(5) When choosing a language to speak with a person who is equally fluent in all your languages, what percentage of time would you choose to speak each language? Please report percent of total time.
(*Your percentages should add up to 100%*):

List language here					
List percentage here:					

(6) Please name the cultures with which you identify. On a scale from zero to ten, please rate the extent to which you identify with each culture. (Examples of possible cultures include US-American, Chinese, Jewish-Orthodox, etc):

List cultures here					
	(click here for scale	(click here for scale	(click here for scale	(click here for scale	(click here for scale

(7) How many years of formal education do you have? ______ ______________________________
Please check your highest education level (or the approximate US equivalent to a degree obtained in another country):

- ☐ Less than High School
- ☐ High School
- ☐ Professional Training
- ☐ Some College
- ☐ College
- ☐ Some Graduate School
- ☐ Masters
- ☐ Ph.D./M.D./J.D.
- ☐ Other:

(8) Date of immigration to the USA, if applicable ___ ______________________________
If you have ever immigrated to another country, please provide name of country and date of immigration here.
______________ ______________________________

(9) Have you ever had a vision problem ☐, hearing impairment ☐, language disability ☐, or learning disability ☐ ? (*Check all* applicable). If yes, please explain (including any corrections):
______________ ______________________________

Language:

This is my (please select from pull-down menu) language.

All questions below refer to your knowledge of .

(1) Age when you…:

began acquiring :	*became fluent* in :	*began reading* in :	*became fluent reading* in :

(2) Please list the number of years and months you spent in each language environment:

	Years	Months
A country where is spoken		
A family where is spoken		
A school and/or working environment where is spoken		

(3) On a scale from zero to ten, please select your *level of* ***proficiency*** in speaking, understanding, and reading from the scroll-down menus:

Speaking	(click here for scale	Understanding spoken language	(click here for scale)	Reading	(click here for scale

(4) On a scale from zero to ten, please select how much the following factors contributed to you learning :

Interacting with friends	(click here for pull-down scale)	Language tapes/self instruction	(click here for pull-down scale)
Interacting with family	(click here for pull-down scale)	Watching TV	(click here for pull-down scale)
Reading	(click here for pull-down scale)	Listening to the radio	(click here for pull-down scale)

(5) Please rate to what extent you are currently exposed to in the following contexts:

Interacting with friends	(click here for pull-down scale)	Listening to radio/music	(click here for pull-down scale)
Interacting with family	(click here for pull-down scale)	Reading	(click here for pull-down scale)
Watching TV	(click here for pull-down scale)	Language-lab/self-instruction	(click here for pull-down scale)

(6) In your perception, how much of a foreign accent do you have in ?

(click here for pull-down scale)

(7) Please rate how frequently others identify you as a non-native speaker based on your accent in :

(click here for pull-down scale)

Language:

This is my (please select from pull-down menu) language.

All questions below refer to your knowledge of .

(1) Age when you…:

began acquiring :	*became fluent* in :	*began reading* in :	*became fluent reading* in :

(2) Please list the number of years and months you spent in each language environment:

	Years	Months
A country where is spoken		
A family where is spoken		
A school and/or working environment where is spoken		

(3) On a scale from zero to ten please select your *level of* ***proficiency*** in speaking, understanding, and reading from the scroll-down menus:

Speaking	(click here for scale)	Understanding spoken language	(click here for scale)	Reading	(click here for scale

(4) On a scale from zero to ten, please select how much the following factors contributed to you learning :

Interacting with friends	(click here for pull-down scale)	Language tapes/self instruction	(click here for pull-down scale)
Interacting with family	(click here for pull-down scale)	Watching TV	(click here for pull-down scale)
Reading	(click here for pull-down scale)	Listening to the radio	(click here for pull-down scale)

(5) Please rate to what extent you are currently exposed to in the following contexts:

Interacting with friends	(click here for pull-down scale)	Listening to radio/music	(click here for pull-down scale)
Interacting with family	(click here for pull-down scale)	Reading	(click here for pull-down scale)
Watching TV	(click here for pull-down scale)	Language-lab/self-instruction	(click here for pull-down scale)

(6) In your perception, how much of a foreign accent do you have in ?

(click here for pull-down scale)

(7) Please rate how frequently others identify you as a non-native speaker based on your <u>accent</u> in :

(click here for pull-down scale)

BIBLIOGRAPHY

Altarriba, J., & Basnight-Brown, D. (2007). Methodological considerations in performing semantic- and translation-priming experiments across languages. *Behavior Research Methods, 39*(1), 1-18.

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory, 4*(5), 527-538.

Ashcraft, M. H., & Fierman, B. A. (1982). Mental addition in third, fourth, and sixth graders. *Journal of Experimental Child Psychology, 33*(2), 216-234.

Ashcraft, M. H., & Stazyk, E. H. (1981). Mental addition: A test of three verification models. *Memory & Cognition, 9*(2), 185-196.

Campbell, J. I., & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, and Goodman (1986). *Journal of Experimental Psychology: General, 117*(2), 204-214.

Campbell, J. I. D. & Graham, D. J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology. Special Issue: Skill, 39*(2), 338-366.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General, 130*(2), 299-315.

Chomsky, N. (1986). Knowledge of language: its nature, origin, and use. New York: Praeger.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1-2), 1-42.

Dehaene, S., & Akhavein, R. (1995). Attention, automaticity, and levels of representation in number processing. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 21*(2), 314-326.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371-396.

Dehaene, S., Cohen, L. (1991). Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia, 29,* 1045-1074.

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16*(3), 626-641.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284*(5416), 970-974.

Dijkstra, T., & van Heuven, Walter J. B. (1998). The BIA model and bilingual word recognition. In A. M. Jacobs (Ed.), *Localist connectionist approaches to human cognition.* (pp. 189-225). Mahwah, NJ US: Lawrence Erlbaum Associates Publishers.

Dijkstra, T., & Van Heuven, Walter J. B. (2002). The architecture of the bilingual word recognition system: From identification to decision. *Bilingualism: Language and Cognition, 5*(3), 175-197.

Duyck, W., & Brysbaert, M. (2002). What number translation studies can teach us about the lexico-semantic organization in bilinguals. *Psychologica Belgica, 42*(3), 151-175.

Duyck, W., & Brysbaert, M. (2004). Forward and backward number translation requires conceptual mediation in both balanced and unbalanced bilinguals. *Journal of Experimental Psychology: Human Perception and Performance, 30*(5), 889-906.

Ellis, N. (1992). Linguistic relativity revisited: The bilingual word-length effect in working memory during counting, remembering numbers, and mental calculation. In R. J. Harris (Ed.), *Cognitive processing in bilinguals.* (pp. 137-155). Oxford England: North-Holland.

French, R. M. (1998). A simple recurrent network model of bilingual memory. *Proc. 20th Annu. Cogn. Sci. Soc. Conf.,* pp. 368-373, Erlbaum.

French, R. M., & Jacquet, M. (2004). Understanding bilingual memory: Models and data. *Trends in Cognitive Sciences*, *8*(2), 87-93.

Frenck-Mestre, C., & Vaid, J. (1993). Activation of number facts in bilinguals. *Memory & Cognition, 21*(6), 809-818.

Green, D. W. (1998). Mental control of the bilingual lexico-semantic system. *Bilingualism: Language and Cognition, 1*(2), 67-81.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79*(4), 329-343.

Hamann, M. S., & Ashcraft, M. H. (1985). Simple and complex mental addition across development. *Journal of Experimental Child Psychology, 40*(1), 49-72.

Kelly, M. K. (2002). Getting ready for school: A cross-cultural comparison of parent and child beliefs about and preparations for entry into first grade in China and the United States. *Dissertation Abstracts International*, *63*(11-B), 5550 (University Microfilms No. 3070347).

Kolers, P. A. (1968). Bilingualism and information processing. *Scientific American,* 218, 78-86.

Kroll, J. F., & de Groot, Annette M. B. (1997). Lexical and conceptual memory in the bilingual: Mapping form to meaning in two languages. In J. F. Kroll (Ed.), *Tutorials in bilingualism: Psycholinguistic perspectives.* (pp. 169-199). Mahwah, NJ US: Lawrence Erlbaum Associates Publishers.

Kroll, J. F., & Stewart, E. (1994). Category interference in translation and picture naming: Evidence for asymmetric connection between bilingual memory representations. *Journal of Memory and Language, 33*(2), 149-174.

Li, P., & Farkas, I. (2002). A self-organizing connectionist model of bilingual processing. In J. Altarriba (Ed.), *Bilingual sentence processing.* (pp. 59-85). Amsterdam Netherlands: North-Holland/Elsevier Science Publishers.

Macnamara, J. (1967). The linguistic independence of bilinguals. *Journal of Verbal Learning & Verbal Behavior, 6*(5), 729-736.

Marian, V., Blumenfeld, H. K., & Kaushanskaya, M. (2007). The language experience and proficiency questionnaire (LEAP-Q): Assessing language profiles in bilinguals and multilinguals. *Journal of Speech, Language, and Hearing Research, 50*(4), 940-967.

Marsh, L. G., & Maki, R. H. (1976). Efficiency of arithmetic operations in bilinguals as a function of language. *Memory & Cognition, 4*(4), 459-464.

McClain, L., & Huang, J. Y. (1982). Speed of simple arithmetic in bilinguals. *Memory & Cognition, 10*(6), 591-596.

McClelland, J. L., & Rumelhart, D. E. (1981). An interactive activation model of context effects in letter perception: I. an account of basic findings. *Psychological Review, 88*(5), 375-407.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44*(1-2), 107-157.

McCloskey, M., & Caramazza, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition, 4*(2), 171-196.

McCloskey, M. & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist, 50*(5), 351-363.

McCloskey, M. & Macaruso, P. (1994). Architecture of cognitive numerical processing mechanisms: Contrasting perspectives on theory development and evaluation. *Cognitive/Current Psychology of Cognition, Special Issue: The neuropsychology of cognitive arithmetic. 13*(3), 275-295.

McCloskey, M., Sokol, S. M., & Goodman, R. A. (1986). Cognitive processes in verbal-number production: Inferences from the performance of brain-damaged subjects. *Journal of Experimental Psychology: General, 115*(4), 307-330.

Miller, K. F., Kelly, M. & Zhou, X. (2005). Learning mathematics in China and the United States: Cross-cultural insights into the nature and course of preschool mathematical development. *In*: Handbook of Mathematical Cognition. Campbell, J. I. D. (Ed.), New York, NY US: Psychology Press, 163-177.

Miller, K. F., Smith, C. M., Zhu, J., & Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: The role of number-naming systems. *Psychological Science, 6*(1), 56-60.

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature, 215*(5109), 1519-1520.

Parkman, J. M., & Groen, G. J. (1971). Temporal aspects of simple addition and comparison. *Journal of Experimental Psychology, 89*(2), 335-342.

Potter, M. C., So, K., von Eckardt, B., & Feldman, L. B. (1984). Lexical and conceptual representation in beginning and proficient bilinguals. *Journal of Verbal Learning & Verbal Behavior, 23*(1), 23-38.

Rumelhart, D. E., & McClelland, J. L. (1982). An interactive activation model of context effects in letter perception: II. the contextual enhancement effect and some tests and extensions of the model. *Psychological Review, 89*(1), 60-94.

Spelke, E. S., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition, 78*(1), 45-88.

Stazyk, E. H., Ashcraft, M. H., & Hamann, M. S. (1982). A network approach to mental multiplication. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 8*(4), 320-335.

Stevenson, H. W., & Lee, S. (1998). An examination of American student achievement from an international perspective. In Ravitch, D. (Ed.), *The state of student performance in American schools.* (pp. 7-52). Washington, DC: Brookings.

Thomas, M. S. C., & van Heuven, Walter J. B. (2005). Computational models of bilingual comprehension. In A. M. B. de Groot (Ed.), *Handbook of bilingualism: Psycholinguistic approaches.* (pp. 202-225). New York, NY US: Oxford University Press.

Vaid, J., & Frenck-Mestre, C. (1991). Incidental memory for format of presentation of number stimuli: Evidence from monolinguals and bilinguals. *Brain and Cognition, 17*(2), 272-284.

Winkelman, J. H., & Schmidt, J. (1974). Associative confusions in mental arithmetic. *Journal of Experimental Psychology, 102*(4), 734-736.

Zbrodoff, N. J., & Logan, G. D. (2005). What everyone finds: The problem-size effect. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition.* (pp. 331-345). New York, NY US: Psychology Press.

VITA

Graduate College
University of Nevada, Las Vegas

Michelle M. Guillaume

Degrees:
Bachelor of Science, Mathematics, 2002
University of New Orleans

Master of Science, Mathematical Sciences, 2005
University of Nevada, Las Vegas

Master of Arts, Psychology, 2008
University of Nevada, Las Vegas

Publications:
Silver, N.C., Steiner, E.T., & **Guillaume, M.M.** (in press). SAMPAL: A program for determining sample sizes for testing and estimating coefficient alpha and for comparing two alpha coefficients. *Applied Psychological Measurement*.

Ashcraft, M.H., & **Guillaume, M.M.** (2009). Mathematical cognition and the problem size effect. In B. H. Ross (Ed.), The Psychology of Learning and Motivation (Vol. 51). Academic Press/Elsevier.

Dissertation Title: Bilingualism and Math Cognition

Dissertation Examination Committee:
Chairperson, Mark H. Ashcraft, Ph.D.
Committee Member, David E. Copeland, Ph.D.
Committee Member, Colleen M. Parks, Ph.D.
Graduate Faculty Representative, CarolAnne M. Kardash, Ph.D.